Measurement of Planck's constant Based on Planck's Radiation Theory

Kwalar B. N.[1] and Adams U.[2]
[1]Department of Physics, Taraba State University, Jalingo, Taraba State.
Email: bnkwalar@yahoo.com. Phone number: 08161581945
[2]Department of Physics, Modibo Adama University of Technology, Yola, Adamawa State.

ABSTRACT

Planck's constant measurement was based on the assumption that electrical power dissipated by the filament which emitted light entirely as radiation and the tungsten filament of the light bulb is a perfect black body. A phototransistor with a fixed frequency υ measured the light intensity emitted over a narrow range of frequencies as an optical filter was employed in a dark room. The intensity of illumination which is analogous to intensity of radiation was deduced from Planck's radiation law and a relationship between the intensity of illumination (or radiation) was therefore established. A power law and an empirical R – T relation were used to calculate the temperatures corresponding to different resistances of the filament as the voltage was varied. The photocurrent was assumed to be proportional to the intensity of radiation and a linear regression fit to the data was used to determine the Planck's constant h, from the slope and the Planck's constant so obtained was 5.1227 x 10^{-34}Js with a percentage accuracy of 13%under laboratory conditions.

Keywords: *Planck's constant, radiation, black body, electrical power and Phototransistor.*

INTRODUCTION

Scientists believed that the main principles of Physics were well known in the late 1800's. There were, however, some phenomena such as blackbody radiation and the photoelectric effect that had not yet been reconciled in a manner consistent with classical theory. At the turn of the nineteenth century the German Physicist Max Planck developed a model explaining black body radiation which claimed that the energy of electric oscillations of atoms could only change by discrete prescribed amounts. He found that a quantum of energy is directly proportional to the frequency of oscillation of the atoms from which the proportionality constant was known as the Planck's constant. It is said that Planck's constant resulted in the appearance of the discontinuity in matter. Actually, the discontinuity discovered by the German Physicist Max Planck does not affect matter but rather interactions and forces. The quantization of physical quantities such as energy and angular momentum, the particle properties of radiation and the wave particles of matter, etc, are directly related to the existence of a universal constant called Planck's constant (Bransden and Joachain, 2000). Just as the velocity, C, of light plays a central role in relativity, so does Planck's constant in Quantum Physics.

Experimental measurement of light emitted through black body radiation, however, did not match predictions made from theory. Max Planck found that he could fit the experimental measurements and theory together by assuming that the vibrating molecules that emitted the light could only have a fixed amount of energy. Instead of energy existing through a continuous range of amounts, Planck found that the vibrating molecules could only have energy in discrete amount of energy or quanta. He thus developed the concept of quantization of energy, that is, molecules involved in black body radiation vibrate with quantized energy, E, according to the relationship $E = nh\upsilon$; where n is a positive integer, υ frequency (Hz) of vibration of molecules, and h is proportionality constant known as Planck's constant (Tannoudji, 1991).

Max Planck, the founder of quantum theory was gravely shocked by his own discovery that energy was not infinitely divisible, but could be transferred only in discrete 'quanta' (Ziman, 1991).His discovery of quantized energy states was a radical revolutionary development and most scientists, including himself did not believe it at the time. He spent time and effort trying to disprove his own discovery, which was eventually to revolutionalize Physics (Tillery, 1991).

Crandall and Delord (1983) proposed the original method that has the advantage of requiring filament area calculation. On the other hand, Dryzed and Ruebenbauer (1991) proposed a rather cumbersome method in which an optical pyrometer is used to determine a calibration curve for the filament resistance dependence upon temperature. The procedure presented by Brizuela and Juan (1996) in which a polyvinyl chloride (PVC) tube blackened inside was used to set up the light bulb and the intensity detector (or phototransistor) in order to avoid any other source of radiation (that is any stray light in the room). This procedure is however less accurate than the one reported by Dryzed and Ruebenbauer (1991), but it is more adequate for reproduction in the laboratory for which the Planck's constant obtained was 4.71E-34 Js which is reasonably close to the accepted value, within an accuracy of 17%. This work therefore seeks to improve on the accuracy of the laboratory work reported by Brizuela and Juan (1996). This work was carried out in a dark room in order to avoid any stray light in the room reaching the phototransistor.

The surface of a hot body emits energy in the form of electromagnetic radiation. This emission occurs at any temperature greater than absolute zero, the emitted radiation being continuously distributed over wavelengths. The distribution in wavelengths or spectral distribution depends on temperature. At low temperature, most of the emitted energy is converted at relatively long wavelengths, such as those corresponding to infrared radiation. As the temperature increases, a large fraction of the energy is radiated at lower wavelengths. The total power (energy per unit time) radiation also increases as the body becomes hotter (Bransden and Joachain, 2000).

When radiation falls on the surface of a body, part of it is reflected and part is absorbed. For example, dark bodies absorb most of the radiation falling on them, while light coloured bodies reflect most of it. The absorption coefficient of a body surface at a given wavelength is defined as the fraction of the radiant energy, incident on the surface, which is absorbed at that wavelength. If a body is in thermal equilibrium with its surrounding, and therefore is at constant temperature, it must emit and absorb the same amount of radiant energy per unit time. The radiation emitted or absorbed under these circumstances is known asthermal radiation (Bransden and Joachain, 2000).

A black body is a body which absorbs all the radiant energy falling upon it. In other words, its absorption coefficient is equal to unity at all wavelengths. Thermal radiation absorbed or emitted by a black body is called black body radiation. Kirchhoff's law states that for any wavelength, the ratio of the emissive power or spectral emittance (defined as the power emitted per unit area at a given wavelength) to the absorption coefficient is the same for all bodies at the same temperature and is equal to the emissive power of the black body at that temperature. Since the maximum value of the absorption coefficient is unity and corresponds to a black body, it follows from Kirchhoff's law that black body is not only the most efficient emitter, but also the most efficient absorber of electromagnetic energy. A perfect black body is an idealization, but it can be very closely approximate in the following way (Pauling and Wilson, 1935).

Consider a cavity kept at a constant temperature whose interior walls are blackened and connected to the outside by a hole. To an outside absorber, a small hole made in wall of such a cavity behaves like a black body surface. The reason is that any radiation incident from the outside upon a hole will pass through it and will almost completely be absorbed in multiple reflections inside the cavity, so that the hole has an effective absorption coefficient close to unity. Since the cavity is in thermal equilibrium, the radiation within it and that escaping from the small opening can thus be closely identified with the thermal radiation from a black body. It should be noted that the hole appears black only at low temperatures, where most of the energy is emitted at wavelengths longer than those corresponding to visible light (Pauling and Wilson, 1935).

We specify the spectrum of black body radiation inside the cavity in terms of a quantity $\rho(\lambda, T)$ which is called the (wavelength) spectral distribution function or (wavelength) monochromatic energy density. It is defined so that $\rho(\lambda, T)d\lambda$ is the energy density (that is the energy per unit volume) of the radiation in the wavelength interval $(\lambda, \lambda + d\lambda)$ at the absolute temperature, T.Wien, Lord Raleigh and J. Jeans derived a spectral distribution function $\rho(\lambda, T)$ from the laws of classical Physics in the following way:

Firstly, from electromagnetic theory, it follows that the thermal radiation within a cavity must exist in the form of standing electromagnetic waves. Secondly, the number of such waves or the number of modes of oscillation of the electromagnetic field in the cavity per unit volume, with wavelengths within the interval λ to $(\lambda + d\lambda)$ can be shown to be $(8\pi / \lambda^4)d\lambda$, so that $n(\lambda) = 8\pi/\lambda^4$ is the number of modes per unit volume and per unit wavelength range. Thirdly, this number is independent of the size and shape of a sufficiently large cavity. Now if $\bar{E}$ is the average energy in the mode with wavelength λ, the spectral distribution function $\rho(\lambda, T)$ is simply the product of $n(\lambda)$ and $\bar{E}$, and hence may be written as

$$\rho(\lambda, T) = \frac{8\pi}{\lambda^4}\overline{E} \quad (1)$$

(Pauling and Wilson, 1935).

THEORY

In December 1900, M. Planck presented a new form of the black body radiation spectral distribution base on a revolutionary hypothesis. He postulated that the energy of an oscillator of a given frequency, υ, cannot take arbitrary values between zero and infinity, but can only take on the discrete values nE_0, where n is a positive integer or zero and E_0 is a finite amount or a quantum of energy, which may depend on the frequency, υ. In this case the average energy $\bar{E}$ of an assemblage of oscillation of frequency, υ, in thermal equilibrium is given by

$$\overline{E} = \frac{\sum_{n=0}^{\infty} nE_0 \exp(-\beta nE_0)}{\sum_{n=0}^{\infty} \exp(-\beta nE_0)}$$

$$= \frac{-d}{d\beta}\left[\log \sum_{n=0}^{\infty} \exp(-\beta nE_0)\right]$$

Using Binomial expansion theorem;

$$\overline{E} = \frac{-d}{d\beta}\left[\log\left\{\frac{1}{1-\exp\left(-\beta E_0\right)}\right\}\right]$$
$$= \frac{E_0}{\exp\left(\beta E_0\right)-1} \qquad (2)$$

(Where $\beta = 1/KT$) assuming, as Planck did that the Boltzmann probability distribution factor can still be used. Substituting for $\bar{E}$ into Eqn. (1) gives

$$\rho(\lambda,T) = \frac{8\pi}{\lambda^4}\frac{E_0}{\exp\left(E_0/KT\right)-1} \qquad (3)$$

E_0 is taken to be proportional to the frequency, υ from the equation;

$$E_0 = h\upsilon = hC/\lambda \qquad (4)$$

Substituting for E_0 in Eqn. (3), the spectral distribution law for $\rho(\lambda, T)$ is thus given by

$$\rho(\lambda,T) = \frac{8\pi hC}{\lambda^5}\frac{1}{\exp\left(hC/\lambda KT\right)-1} \qquad (5)$$

Equation (5) is known as Planck's radiation law (Bransden and Joachain, 2000).

According to classical electromagnetic theory, a system of accelerated electrically charged particles emits radiant energy. In a bath of radiation at temperature, T, it also absorbs radiant energy, the rates of absorption and emission being given by classical laws. These opposing processes might be expected to lead to a state of equilibrium. The following treatment of the corresponding problem for quantized systems (atoms or molecules) was given by Einstein's in 1916 as mentioned earlier(Pauling and Wilson, 1935).Consider an enclosure, containing atoms (of a single kind) and radiation in thermal equilibrium at an absolute temperature, T. Let a, and b, denote two atomic states of energies E_a and E_b respectively, with $E_a \langle E_b$, where a, is a state with lower energy and, b, is a state with higher energy. The state a, and b, are assumed to be non-degenerate. Non-degenerate here means a system for which the number of quantum states g_i is equal to one, i.e., each energy level may be attained in only one way (Kittle and Kroemer, 1980). Transition from one state is always accompanied by the emission or absorption of radian energy. According to Bohr frequency rule, the number of atoms per unit time, N_{ba}, making a transition from a to b by absorbing radiation of frequency υ, given by

$$\upsilon = \upsilon_{ba} = \frac{\left(E_b - E_a\right)}{h}$$

Where h is the Planck's constant is proportional to the total number, N_a , of atoms in the state, a, and also to the energy density of the radiation per unit frequency range $\rho(\upsilon_{ba})$. Thus we have

$$N_{ba} = B_{ba}N_{a\rho}(\upsilon_{ba})$$

Where B_{ba}, is called the Einstein Coefficient of absorption (Holman, 1988).The probability of absorption of radiation is thus assumed to be proportional to the density of radiation. On the other hand, it is necessary to postulate that the probability of emission is the sum of two parts; one of which is independent of the radiation density and the other proportional to it. It should however be noted that this postulate is analogous to the classical theory, according to which an oscillator interacting with an electromagnetic wave could either absorb energy from the field or lose energy to it depending on the relative phases of the oscillator and wave (Pauling and Wilson, 1935).We therefore assume that the number of atoms making the transition from state, b, to state, a, per unit time is independent of the radiation density ρ, considering one part, and the number of stimulated (or induced) transitions per unit time is proportional to radiation density, ρ, considering another part.

Thus

$$N_{ab} = A_{ab}N_b + B_{ab}N_b\rho(\upsilon_{ba}) \quad (8)$$

Where A_{ab} is the Einstein coefficient for spontaneous emission, and B_{ab} is the Einstein coefficient of stimulated or induced emission. Stimulated or induced emission corresponds to a transition in which the atom loses energy so that $E_b < E_a$ and $\upsilon \approx -\upsilon_{ba} \approx (E_a - E_b)/h$. it should be noted that under the same radiation field, the number of transition per second exciting an atom from state, a, to the state, b, is the same as the number de-exciting the atom from state, b, to state, a. The thermo dynamical principle of detailed N_{ba} balancing states that in an enclosure containing atoms and radiation in equilibrium, the transition rate from a to b, N, is the same as that from b to a, N_{ab}, where a and b are any pair of states. Since at equilibrium, $N_{ba} = N_{ab}$, we equate Eqns (2) and (3) to get

$$B_{ba}N_a\rho(\upsilon_{ba}) = A_{ab}N_b + B_{ab}N_b\rho(\upsilon_{ba})$$

$$= N_b\left[A_{ab} + B_{ab}\rho(\upsilon_{ba})\right]$$

$$\Rightarrow \frac{N_a}{N_b} = \frac{A_{ab} + B_{ab}\rho(\upsilon_{ba})}{B_{ba}\rho(\upsilon_{ba})} \quad (9)$$

From a standard result in thermodynamics, it is known in thermal equilibrium that the ratio N_a/N_b is given by

$$\frac{N_a}{N_b} = \exp\{-(E_a - E_b)/KT\} \quad (10)$$

From Eqn. (1)

$$E_b - E_a = h\upsilon_{ba}$$

$$\Rightarrow E_a - E_b = -h\upsilon_{ba}$$

Substituting for $(E_a - E_b)$ in Eqn. (10) gives

$$\frac{N_a}{N_b} = \exp(h\upsilon_{ba}/KT) \quad (11)$$

Where K is Boltzmann's constant. Combining Eqns. (9) and (11) gives

$$\frac{A_{ab} + B_{ab}\rho(\upsilon_{ba})}{B_{ba}\rho(\upsilon_{ba})} = \exp(h\upsilon_{ba}/KT)$$

$$\Rightarrow B_{ba}\rho(\upsilon_{ba})\exp(h\upsilon_{ba}/KT) = A_{ab} + B_{ab}\rho(\upsilon_{ba})$$

$$\Rightarrow \{B_{ab}\exp(h\upsilon_{ba}/KT) - B_{ab}\}\rho(\upsilon_{ba}) = A_{ab}$$

$$\rho(\upsilon_{ba}) = \frac{A_{ab}}{B_{ba}\exp(h\upsilon_{ba}/KT) - B_{ab}} \quad (12)$$

An alternative expression for the energy density per unit wavelength λ of interval ρ(λ, T) at temperature, T, is given by Planck's radiation law as

$$\rho(\lambda,T) = \frac{8\pi hC}{\lambda^5}\frac{1}{\exp(hC/\lambda KT) - 1} \quad (13)$$

(from Eqn. (15) where C is the speed of light). Writing for simplicity, $\rho(\lambda) \equiv \rho(\lambda,T)$, we have the relation

$$\rho(\upsilon) = \rho(\lambda)\left|\frac{d\lambda}{d\upsilon}\right| \quad (14)$$

It is known that

$$\lambda = \frac{C}{\upsilon}$$

$$\Rightarrow \frac{d\lambda}{d\upsilon} = -\frac{C}{\upsilon^2}$$

Substituting for $d\lambda/d\upsilon$ in Eqn. (14) gives

$$\rho(\upsilon) = \upsilon(\lambda)\frac{C}{\upsilon^2} \quad (15)$$

Knowing that $\upsilon = \upsilon_{ba}$, then $\lambda = \lambda_{ba}$, thus $C = \lambda_{ba}\upsilon_{ba}$ and $\lambda_{ba} = C/\upsilon_{ba}$. Hence substituting for λ, υ and ρ(λ,T) from Eqn. (13) in Eqn. (15) gives

$$\rho(\upsilon_{ba}) = \frac{8\pi hC}{C^5/\upsilon_{ba}^5}\frac{1}{\exp\left(hC/\frac{C}{\upsilon_{ba}}KT\right)^{-1}}\cdot\frac{C}{\upsilon_{ba}^2}$$

$$= \frac{8\pi hC^2\upsilon_{ba}^5}{C^5\upsilon_{ba}^2}\frac{1}{\exp(h\upsilon_{ba}/KT) - 1}$$

$$= \frac{8\pi h\upsilon_{ba}^3}{C^3}\frac{1}{\exp(h\upsilon_{ba}/KT) - 1} \quad (16)$$

Eqn. (16) is Planck's radiation law. Comparing Eqn. (16) with Eqn. (12) we have

$$B_{ab} = B_{ba} \quad (17)$$

and

$$A_{ab} = \frac{8\pi h\upsilon_{ba}^3}{C^3}B_{ab} \quad (18)$$

From Eqns. (17) and (18), it is deduced that the coefficient of absorption and stimulated or induced emission are equal and the coefficient of spontaneous emission differs from them by the factor $8\pi h\upsilon_{ba}^3/C^3$. At the temperature, T = $h\upsilon_{ba}$/Klog2, the probabilities of spontaneous emission and induced emission are equal (Bransden and Joachain, 2000).

DEDUCTION OF INTENSITY OF ILLUMINATION FROM PLANCK'S RADIATION LAW

From Planck's radiation law (i. e. Eqn. (16)), $\rho(\upsilon, T)$ the energy density of the radiation per unit frequency range is given by

$$\rho(\upsilon,T) = \frac{8\pi h}{C^3}\frac{\upsilon^3}{\exp(h\upsilon/KT)-1} \qquad (19)$$

Unit of $\rho(\upsilon, T\}$ is Jsm^{-3}.But intensity of illumination which is analogous to intensity of electromagnetic radiation (having units Wm^{-2} or $Jm^{-2}s^{-1}$) is the luminous flux per unit area called illuminance (Jewett and Serway, 2005). Unit of illuminance or intensity of illumination (or radiation), $I(\upsilon,T)$ is $Jm^{-2}s^{-1}$.We know that units of $\rho(\upsilon, T)$ is

$$\frac{Js}{m^3} = \frac{J}{sm^2}\cdot\frac{s^2}{m}$$
$$= \frac{J}{sm^2}\cdot\frac{1}{(ms^{-1})(s^{-1})}$$

Since ms^{-1} is the unit of speed of light, C, while s^{-1} is the unit of frequency, υ, and $Jm^{-2}s^{-1}$ being unit of intensity of radiation (or illumination), $I(\upsilon, t)$, then from Planck's radiation law,

$$\rho(\upsilon,T) = I(\upsilon,T)\cdot\frac{1}{C\upsilon} = \frac{8\pi h}{C^3}\frac{\upsilon^3}{\exp(h\upsilon/KT)-1}$$

$$\Rightarrow I(\upsilon,T) = C\upsilon\rho(\upsilon,T) = \frac{8\pi h}{C^2}\frac{\upsilon^4}{\exp(h\upsilon/KT)-1} \qquad (20)$$

Where T is the absolute temperature, C is the speed of light, K is Boltzmann's constant, and h is the Planck's constant. Eqn. (20) can be utilized by using a single frequency. The ratio of intensities I_1 and I_2 measured at the same frequency and at two different temperatures T_1 and T_2, is expressed as follows:

$$\frac{I_1(T_1)}{I_2(T_2)} = \frac{\exp(h\upsilon/KT_2)-1}{\exp(h\upsilon/KT_1)-1} \qquad (21)$$

It is helpful to use a relationship between R and T to avoid direct filament temperature measurements. This relationship is achieved empirically. Hence voltage V and current I measurements of the light bulb obtained are used to calculate the filament resistance, R, and power, P, dissipated. That is, for a black body the emitted power, P, is given by Stefan's law as follows;

$$P = A\sigma T^4 \qquad (22)$$

σ is Stefan's constant and A is area of the emitter (that is the filament). Assuming a power law $T\alpha R^{\gamma}$, the power dissipation P can be written as

$$P = I^2R = A\sigma T^4 = CR^{4\gamma} \qquad (23)$$

The Eqn. (24) below also is used to get resistance R and power P of the bulb as black body used.

$$R = V/I; \quad P = I^2R \qquad (24)$$

Where C = $A\sigma$ = constant. Applying natural logarithm on both sides of Eqn. (23) we have

$$\ln P = \ln C + 4\gamma \ln R \qquad (25)$$

Then the empirical R – T relation is given by

$$T = \left(\frac{R}{R_0}\right)^{\gamma} T_0 \qquad (26)$$

Where R_0 is the resistance of the bulb filament at room temperature T_0 and γ is the power obtained from the plot of $\ln P$ against $\ln R$ (i. e. using Eqn. (25)) (Brizuela and Juan, 1996). Eqn. (21) gives

$$\frac{I_1(T_1)}{I_2(T_2)} = \frac{\exp(h\nu/KT_2)-1}{\exp(h\nu/KT_1)-1} \qquad (27)$$

When $h\nu\rangle\rangle KT$, then $\exp(h\nu/KT)\rangle\rangle 1$, so that Eqn. (27) can now be expressed as follows:

$$\frac{I_1(T_1)}{I_2(T_2)} = \frac{\exp(h\nu / KT_2)}{\exp(h\nu / KT_1)} \qquad (28)$$

$$\Rightarrow \frac{I_1(T_1)}{I_2(T_2)} = \exp\left(T_2^{-1} - T_1^{-1}\right)\frac{h\nu}{K} \qquad (29)$$

Applying natural logarithm on both sides of Eqn. (29) gives

$$\ln\left(\frac{I_1}{I_2}\right) = \frac{h\nu}{K}\left(T_2^{-1} - T_1^{-1}\right) \qquad (30)$$

Where $\nu = \frac{C}{\lambda}$ (from $C = \lambda\nu$), C = 3 X 10^8m/s and K = 1.381 x 10^{-23}JK^{-1}. Simplification of Eqn. (27) directly gives

$$I_1(T_1)\exp\left(\frac{\nu}{KT_1}\right)h - I_2(T_2)\exp\left(\frac{\nu}{KT_2}\right)h + \{I_2(T_2) - I_1(T_1)\} = 0$$

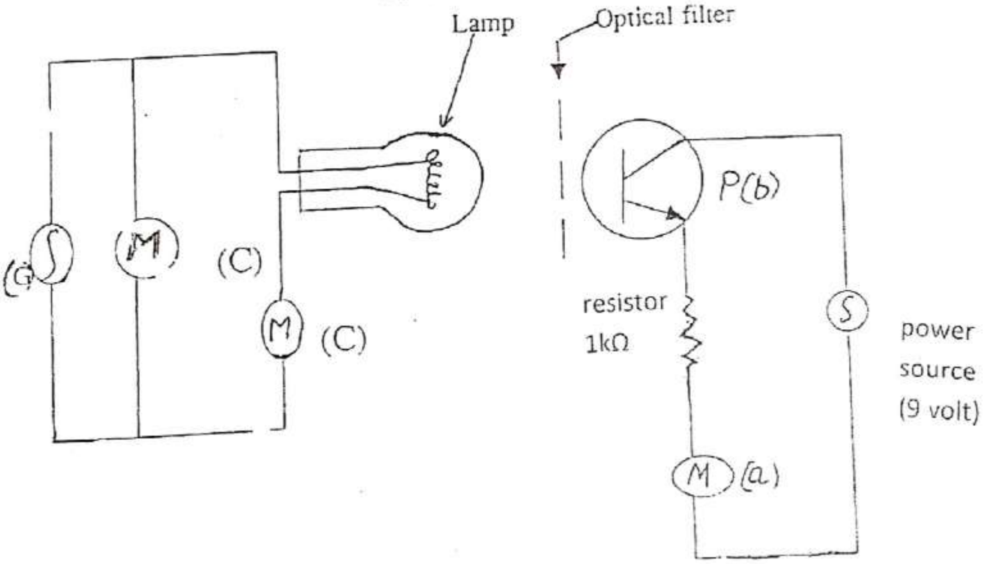
If photocurrent is assumed to be proportional to the intensity of radiation (Brizuela and Juan, 1996), then Eqn. (28) becomes

$$\ln\left(\frac{I_1(T_1)}{I_2(T_2)}\right) = \frac{h\nu}{K}\left(T_2^{-1} - T_1^{-1}\right)$$

The first temperature T_1 and the corresponding photocurrent I_1 is taken as reference. Then $\ln(I_1/I_2)$ is plotted against $\left(T_2^{-1} - T_1^{-1}\right)$ and a linear regression fit to the data is used to determine h, from the slope. One particular line along which all points lie on or near it is called regression line. It is also called least square line or line of best fit and this concept is based on the gradient of a straight line (Adegun, 1992). (31)

EXPERIMENTAL SET – UP

The pieces of apparatus used in these experiments consist of commercially available incandescent lamp (60 W – 220 V), bulb socket, an optical filter of wavelength 546 nm, resistor (1.0 KΩ), two digital multimeters, a micro ammeter, a thermometer, LCR – meter, retort stand and clamp, general purpose power supply (0 – 350 V), power source and auto-transformer. The experiment was carried out in a dark room. It is the selective ability of the optical filter to absorb selectively light of a particular frequency that it is found useful in this experimental work. The experimental set-up and electric circuit is shown Fig. 1.



(G) General purpose supply (0-350V)
(a) Microammeter
(b) Phototransistor
(C) Multimeter

Fig. 1. Experimental set-up and electric circuit

PROCEDURE

Commercially available incandescent lamps (60 W-220 V) were used as sources of black body radiation. A single colour optical filter was used, with wavelength of 546 nm and a frequency in the visible range (5.4945E 14 S^{-1}). The optical filter was interposed in order to have the phototransistor irradiated essentially by a 'fixed' frequency υ. A simple electrical set-up which included an autotransformer was used for voltage variations. As the AC voltage was increased, the resistance R and temperature T of the filament was also increased. The initial temperature T_o and initial resistance R_o were measured and the experiment was then performed using a thermometer and LCR-meter. R_o was the resistance of the filament measured at room temperature T_o. Another quantity to be measured was the emitted light intensity from the incandescent lamp. In this work, the experiment was carried out in a dark room in order to avoid any other source of radiation (that is any stray light in the room). As shown in Figure 3, a bulb socket was fixed on one side of the optical filter with the help of a clamp while a phototransistor was fixed on the other with the help of a retort stand and clamp. The resulting light intensity that reached the phototransistor generated a photocurrent that was measured with a micro ammeter as a voltage drop across the resistor (1KΩ).An appropriate polarization voltage or bias voltage of 9V was applied to the phototransistor to assure different intensities of a given pair of illumination powers. The applied dc voltage depends on the characteristic V – I of the phototransistor used. The sequences of photocurrent measurements were then repeated for a second setting of lamp voltage which in turn resulted in another filament temperature. Digital multimeters were used to measure the voltage and current of the light bulb filament whereas the emitted light intensity was determined with a phototransistor at a fixed frequency υ.

RESULTS AND DISCUSSION

Using Eqns. (24) and (26), experimental data and calculated resistance, power and temperature are given in Table 1. The empirical power γ was determined from a linear regression fit to the logarithmic P-R plot as shown in Figure2, whose slope is equal to 4γ using Eqn. (25).

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Table 1. Experimental data, calculated resistance, power and temperature.

$R = V/I$; $P = I^2R$ **(λ = 546 nm; $T_0 = 36^0$ C= 309 K; R_0 = 74.6 Ω)**

V/ volt(V)	I/A	R/Ω	P/W	lnR	lnP	T/K	I_p/mA
110	0.170	647.0588	18.7000	6.4724	2.9285	1572.3450	0.01
120	0.183	655.7377	21.9600	6.4858	3.0892	1588.3009	0.02
140	0.195	717.9487	27.3000	6.5764	3.3069	1701.2269	0.03
160	0.200	800.0000	32.0000	6.6846	3.4657	1846.6146	0.04
190	0.225	844.4444	42.7500	6.7387	3.7554	1923.8456	0.08
210	0.238	882.3529	49.9800	6.7826	3.9116	1988.9433	0.15
230	0.250	920.0000	57.5000	6.8244	4.0518	2052.9250	0.24
250	0.260	961.5385	65.0000	6.8685	4.1744	2122.7889	0.38
270	0.269	1003.7175	72.6300	6.9115	4.2854	2192.9860	0.53
290	0.280	1035.7143	81.2000	6.9428	4.3969	2245.7609	0.76
300	0.286	1048.9510	85.8000	6.9555	4.4520	2267.4773	0.99

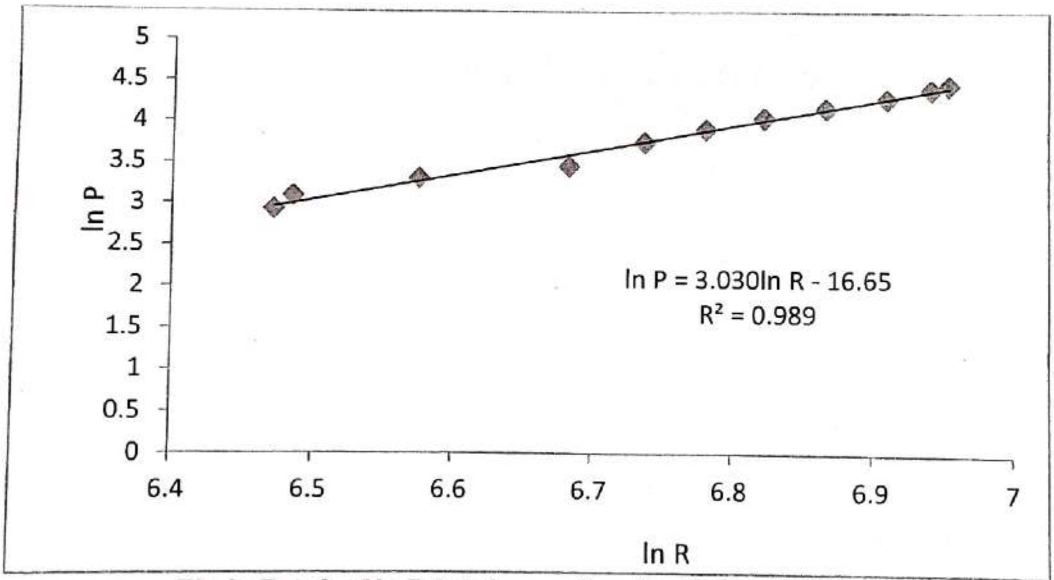

Fig.2. Graph of ln P (total power) against ln R; γ =0.7578

Planck's constant h was obtained with any pair of temperatures and the associated pair of photocurrents by applying Eqn. 27. However, the first temperature T_1 = 1572.345 K and the corresponding photocurrent I_1 = 0.01 mA was taken as a reference temperature and photocurrent respectively as recorded in Table 2. Then the quantity $\ln (I_1/I_2)$ was plotted against the quantity $(1/T_2 - 1/T_1)$, as shown in Fig. 2. Using Eqn. 30, h was then determined from a linear regression fit to the $\ln (I_1/I_2)$ - $(1/T_2 - 1/T_1)$ plot, whose slope was equal to $h\upsilon/K$ ($=hC/K\lambda$).

Table 2. Data of reference Temperature T_1 = 1572.345 K and the corresponding reference Photocurrent I_1 = 0.01 mA for which ratio of I_1 to other Photocurrents I_2 and the difference between T_1^{-1} and other inverse temperatures T_2^{-1} are computed.

I_2/mA	I_1/I_2	T_2/K	T_2^{-1}/K^{-1}	$\ln(I_1/I_2)$	$(T_2^{-1}-T_1^{-1})/K^{-1}$
0.02	0.5000	1588.3009	6.2960E-04	-0.6931	-6.3864E-06
0.03	0.3333	1701.2269	5.8781E-04	-1.0987	-4.8199E-05
0.04	0.2500	1846.6146	5.4153E-04	-1.3863	-9.4458E-05
0.08	0.1250	1923.8456	5.1979E-04	-2.0794	-1.1620 E-04
0.15	0.0667	1988.9433	5.0278E-04	-2.7076	-1.3321E-04
0.24	0.0417	2052.9250	4.8711E-04	-3.1773	-1.4888E-04
0.38	0.0263	2122.7889	4.7108E-04	-3.6382	-1.6491E-04
0.53	0.0189	2192.9860	4.5600E-04	-3.9686	-1.7999E-04
0.76	0.0132	2245.7609	4.4528E-04	-4.3275	-1.9071E-04
0.99	0.0101	2267.4773	4.4102E-04	-4.5952	-1.9497E-04

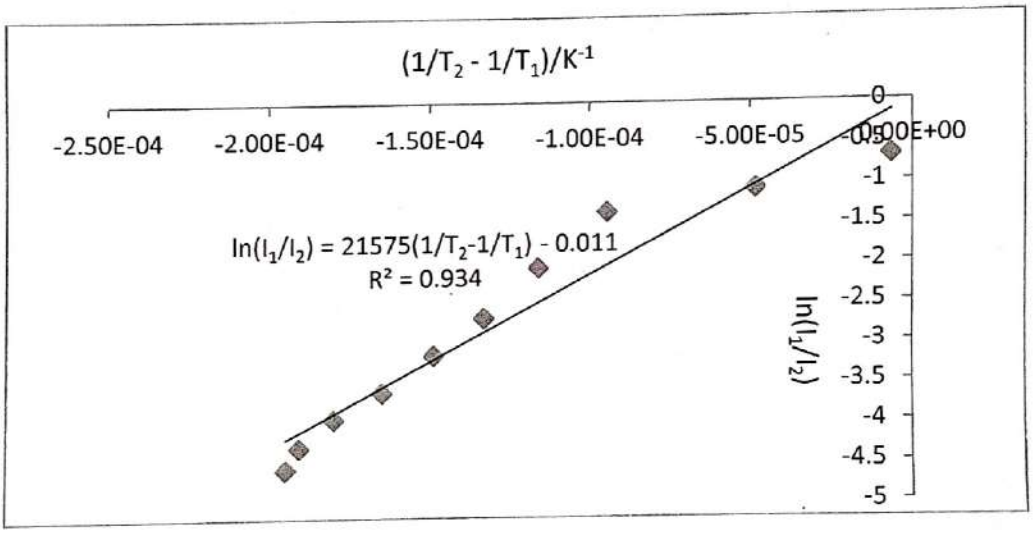

Fig. 3. Graph of ln (I_1/I_2) against $(1/T_2 - 1/T_1)$

The Planck's constant h so obtained was 5.1227E-34 Js from the slope in Fig. 3with a standard error of coefficient of 2.8E-35 Js. Any other temperature taken as a reference will yield a similar value for h in the range (5.1 – 6.0) E-34 Js. At large $(1/T_2 - 1/T_1)$, the plotted points varied from linearity because a saturation photocurrent appeared in the phototransistor at higher filament temperature.

CONCLUSION

The value obtained for Planck's constant h was 5.1227E-34 Js. It was reasonably close to the accepted value, within an accuracy of 13%, and thus statistically significant. Hence there was an improvement in the results obtained in this study. Some of the sources of discrepancy were the ignored convection loss corrections for glass filled lamps, some heat energy that heated up the glass itself and the fact that a light filament was not a perfect black body.

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