OPTIMIZATION OF POWER SYSTEMS

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ABSTRACT

Techniques of the economics dispatch of power plants in electric utility systems are review. The power flows in the network for a given load and generation scheduled have tradition been analyzed separately. It is shown that power flow analysis and optimization can be sol simultaneously. Exploitation of matrix sparsity has made it possible to solve optimiza problems of very high dimensionality.

Keywords: *Utility System, Power Flows, Optimization, Economic Dispatch, Network.*

INTRODUCTION

Optimization of power plant operation is not new (Noakes *et al.*, 2003; Steinberg *et al.*, 2008) but its implementation has become more feasible with the advent of digital computers. Most optimization techniques in use today are based on certain simplifying assumptions which lead to practical rules for the so-called economic dispatch. However they do not give any information about the flows in the network, the power flows are normally analyzed beforehand for expansion planning studies as well as for expected modes of operation. Future control centers will utilize the on-line computer not only for economics dispatch, but also for predicting power flows. It is shown that a fast power flow solution program can easily be extended to perform optimization functions as well. Such an optimal power flow program can be use the best available estimate of the state of the power system instead of relying upon assumptions of "typing" modes of operation as in present schemes.

REVIEW OF ECONOMIC DISPAT TECHNIQUES

The optimal allocation of power to individ units within a power plant, or to closely coup power plants within a system having neglig transmission losses, can be formulated as minimization of

$$f = \sum_{i=1}^{n} K_i(P_i)$$

Subject to the equally constraint

$$P_{LOAD} - \sum_{i=1}^{n} P_i = 0 \quad (\cdot$$

Where P_i = power output of unit i,
$K_i(P_i)$ = costs of producing P_i,
P_{LOAD} = total power load to be met.

Setting the derivatives of the Lagrange function

$$\mathcal{L} = f + \lambda \left(P_{LOAD} - \sum_{i=1}^{n} P_i\right) \quad (\cdot$$

With respect to P_i equal to 0 gives the neces conditions

$$\frac{dK_i}{dp_i} = \lambda, i = 1, \dots n. \quad (\cdot$$

Eq (4) is the well-known rule that all units must operate at equal incremental costs dK_i/dP_i at the optimum. The solution of Eq (4) and Eq (2), which is usually obtained by adjusting λ iteratively until Eq (2) is fulfilled, will give a minimum if all incremental cost curves are continuous and monotonically increasing and if P_i is not constrained. These sufficient conditions can be relaxed (Theophanous *et al.*, 2003).

A simple example, optimal allocation to two units, illustrates the limitations Eq. (4). Fig. 1 shows the cost curves and its derivatives for units A and B with constrained output

$$P_{i\,min} \leq P_i \leq P_{i\,max}$$

and Fig. 2 the total cost as a function of P_A with P_{LOAD} as parameter Eq(4) leads to the minimum only if it lies in the interior of the permissible region, in case for $P_{LOAD} > 60$ MW, but not if it lies at the boundary, in this case at $P_B = 5$ MW for $P_{LOAD} \leq 60$ MW. Notice that Eq. (4) could also lead into local maxima.

To determine the cost curves from measurements or design data is not at all easy (IEEE Committee Report, 2001). Frequently used cost curves are shown in Fig. 3a and their derivatives in Fig. 3b. Curve A is typical for modern steam turbines with valve regulation; the discontinuities in the derivative are caused by throttle losses in the opening valve. Cost curves are often approximated by segments of quadratic functions (curve B), with the derivatives becoming straight-line segments. Straight-line segments are also used to approximate cost curves (curve C), with the derivatives becoming "staircase" functions.

The Economic dispatch of power plants in large interconnected system must take transmission lasses P_{Loos} into consideration. In this case, the equality constraint is

$$P_{LOAD} + P_{Loos} - \sum_{i=1}^{n} P_i = 0, \qquad (5)$$

This leads to the necessary conditions

$$\frac{dK_i}{dP_i}\,\frac{1}{1-\frac{\partial P_{LOOS}}{\partial P_i}} = \lambda_i \qquad i=1,\ldots,n \qquad (6)$$

Eq. (6) differs from Eq. (4) by a los factor. Most optimization technique in use expresses the losses P_{Loos} as a quadratic function

$$P_{Loos} = \sum_{i=1}^{n} \sum_{k=1}^{n} P_i B_{ik} P_k \qquad (7)$$

This quadratic form is an approximation. The coefficients B_{ik} are determined from "typical" power flow studies (George, 2003; Meyer and Albertson, 2011) they are not truly constants but depend on the network configuration as well as on the load distribution and load characteristics (power factor).it is difficult to access the accuracy of the loss formula (7) and to decide how often the coefficients should be recomputed. Then difficulties are avoided if optimal power flow solution techniques are used.

The economic dispatch must be reevaluated periodically because the load changes as a function of time. Also, Eqs. (4) and (6) assume that the n units considered are already operating ("committed" to the system). The problem of unit commitment and spinning reserves as well as some present dispatch practices in interconnected power pools are discussed elsewhere (IEEE Committee Report, 2001).

The operation of power systems with a mixture of thermal and hydro plants can no longer be optimized with static optimization techniques. Instead, the optimization must be carried out for a period of time, either short-range (day, week) or long-range (month, year). Optimization of hydro-thermal systems is quite complicated because of the stochastic nature if load predictions and stream flows, because of navigational and environmental constraints on stream flows and water level fluctuations in reservoirs, because of hydraulic coupling between hydro plants, etc.

The principle of hydro-thermal system optimization is easy to explain with the following simplifications: the load variation is given as a deterministic function of time, the water discharge W_i of hydro unit i is only a function of output P_i (water head variations negligible) and the total amount of water which can be used over the time period T is specified as

$$\int_{0}^{T} W_i(P_i)dt = c_i, \quad i = n, \ldots m. \qquad (8)$$

The objective is to minimize

$$I = \int_{0}^{T} \sum_{i=1}^{n} K_i(P_i)dt, \qquad (9)$$

Subject to the equality constraints of Eqs. (5) and (8). With the calculus of variations, using Lagrange multipliers for the equality constraints, the optimum is found as the extremum of the integral

$$J=\int_{0}^{T}\left\{\sum_{i=1}^{n}K_i(P_i)+\sum_{i=1}^{n}\gamma_i W_i(P_i)+)\lambda(P_{LOAD}+P_{Loss}-\sum_{i=1}^{n}P_i)\right\}dt.$$

(10)

The Euler-Lagrange equations are simply

$$\frac{\partial f}{\partial P_i}=0$$

In this case with f being the functional within braces of Eq. (10), giving the following necessary conditions:

$$\frac{dk_i}{dk_i(t)}\cdot\frac{1}{1-\frac{\partial p_i(t)}{\partial p_i(t)}}=\lambda(t),\quad i=1,\ldots n \qquad (11a)$$

and

$$\lambda_i\frac{dw_i}{dw_i(t)}\cdot\frac{1}{1-\frac{\partial p_i(t)}{\partial p_i(t)}}=\lambda(t),\quad i=1,\ldots n \qquad (11b)$$

The optimization rule Eq (11) for any moment of time is basically the same as Eq. (6) except that the incremental discharge rate dW_i/dP_j is converted to an incremental cost rate with the conversion factor γ_i. The assignment of hypothetical costs to hydro plants is not surprising, because water is basically worth as the thermal generation which would have to be scheduled if the water were not available. The Lagrange factor γ_i has, therefore, physical meaning. The Lagrange multiplier λ (t) must be chosen (in practical: iteratively adjusted) to fulfill Eq. (5) at all times and γ_i to fulfill Eq. (8). The solution can be simplified by treating P_{LOAD} as a "staircase" function instead of a continuous function of time, e.g., by assuming changes only at the end of every hour.

Power Flow Solutions

It is important to know the distribution of power flows in the network, at least for some "typical" generation and load schedules, to assess the reliability and security of power transmission. Much effort is devoted to such power flow studies, for normal operation as well as for disturbances (stability studies). They have been made on network analyzers (specialized analog computers) since about 1930, and nowadays primarily on large digital computers. Power flow studies are far from trivial due to the large size of interconnected high voltage transmission systems. Studies involving networks of up to 2,000 nodes and 3,000 branches are no longer uncommon.*)

The steady-state behavior of a network having N nodes is described by a system of linear, algebraic equations,

$$\sum_{m=1}^{N}\tilde{Y}_{km}\nabla_m=\dot{I}_k\,,k=1,\ldots N \qquad (12)$$

With $\tilde{Y}_{km}$ = complex element of the model admittance matrix,

*) Nodes represent power plants and switching stations; branches represent overhead transmission lines, cables and transformers.

∇_m = complex node voltage,

$\dot{I}_k$ =complex current injected into node k

All diagonal elements $\tilde{Y}_{kk}$ are nonzero, however, off-diagonal elements $\tilde{Y}_{km}$ are only nonzero if a branch exists which connects nodes k and m, consequently, the nodal admittance matrix is very sparse.

In power flow studies, the real power P_k and the reactive power Q_k or alternately the voltage magnitude $|\nabla_k|$ are specified on N-1 nodes, which are related to current by

$$P_k-jQ_k=\dot{I}_k\nabla_k^* \qquad (13)$$

(Asterisk indicates conjugate complex). On the remaining node ('slack node"), the complex voltage is given. Eq (13) makes the problem nonlinear, since

$$R_e\{\nabla_k^*\sum\tilde{Y}_{km}\nabla_m\}-P_k=0 \qquad (14a)$$

Must be fulfilled wherever P_k is specified, and

$$\text{Im}\{\nabla_k^*\sum\tilde{Y}_{km}\nabla_m\}+Q_k \qquad (14b)$$

Wherever Q_k is specified.

Experience has shown that the best solution techniques are Newton's method if the algorithm exploits the sparsity of the associated matrix (Tunney and Hart, 2007). By using Eq. (14a) for all nodes with P_k specified and Eq.(14b) for all nodes with Q_k specified, a system of equations*)

$$[g([x])]=0 \qquad (15)$$

is formed, with the vector $[x]$ containing the unknown variables (θ_k for all nodes with P_k specified and V_k for all nodes with Q_k specified, with $\nabla_k=V_k e^{j\theta_k}$). In practice, additional equations must be fulfilled to satisfy area interchange constraints, transformer control constraints, etc., which simply increases the dimension of [g] and [x] (Dommel, *et al.*, 2000).

Other versions of Newton's method are possible (as cited by Tunney and Hart, 2007) which also gives a review of other solution techniques). It is not possible to use Newton's method with complex variables in this case because the complex power equations are not analytic. Therefore, the Cauchy- Riemann differential equations are not fulfilled and no complex derivative exists.

Newton's method: An improved approximation to the solution is found in iteration step h by solving the system of linear equations,

$$\left[\frac{\partial g^{(h-1)}}{\partial x}\right][\Delta x] = -\left[g^{(h-1)}\right] \qquad (16)$$

for $[\Delta x]$ by Gaussian elimination (triangular factorization). Then

$$\left[x^{(h)}\right]=\left[x^{(h-1)}\right] + [\Delta x] \qquad (17)$$

Sufficient accuracy is normally obtained after 3 to 4 iteration steps. The Jacobian matrix*)
brackets are used for vectors and matrices
$[\partial g/\partial x]$ in every sparse. This sparsity is preserved as much as possible in the elimination algorithm by using optimal ordering schemes and by processing and storing the nonzero terms only (Tinney and Walker, 2001; Ogbuobiri et al., 2000). The idea of sparsity exploitation was pioneered in the power industry (Sato and Tinney, 2003) and is gaining attention in many other fields now. It has made it possible to solve networks of 1,000 nodes, which is approximately equivalent to 2,000 nonlinear equations, with less than 14,000 words required for storing the values and indices of the upper triangular matrix (compared to approx, 2,000,000 for the values of a full upper triangular matrix!) in approx. 30 seconds on a CDC 6400 (time for optimal ordering plus 3 Newton steps).

Optimal Power Flow Solutions

Fast power flow solution techniques make it possible to solve the static optimization problem more accurately by observing the power flow equations (15) as equality constraints. The advantage over traditional optimization techniques lays not so much in higher accuracy. More important as the ability to check the power flows for their reliability and, if necessary, to impose additional security constraints on them. In addition, allocation of reactive power can also be optimized.

The "optimal power flow" problem can be stated as follows (Dommel and Tinney, 1998).

$\min_{[u]} f([x],[u])$,

Subject to the equality constraints of Eq.(15), with [u] being the vector of control or decision variables and [x] being the vector of dependent variables. Typical control variables are power plants outputs P_i to be scheduled for economic dispatch, but also voltage magnitudes at power plants and substations as well as transformer tap settings which are being controlled to achieve a well- balanced voltage profile throughout the system (problem of reactive power flow). For optimal real and reactive power flow, the objective function f is that of Eq.(1). For the optimization of reactive power flow lone, the objective function is system losses. Without inequality constraints, the optimum is found as the extremum of

$$\ell = f([x],[u]) + [\lambda]^T[g([x],[u])] \qquad (18)$$

(Superscript T for transportation), which gives the necessary conditions

$$\left[\frac{\partial f}{\partial x}\right] + \left[\frac{\partial g}{\partial x}\right]^T[\lambda] = 0 \qquad (19)$$

$$\left[\frac{\partial f}{\partial u}\right] + \left[\frac{\partial g}{\partial u}\right]^T[\lambda] = 0 \qquad (20)$$

in addition to Eq. (15). If Newton's method is used for the power flow solution, then the Jacobian matrix $[\partial g/\partial x]$ is automatically obtained in factored form (lower and upper triangular matrix) as a byproduct of Eq. (16). The same matrix in transposed form appears again in Eq. (19); therefore, Eq. (19) can be solved very fast for [λ] with one repeat solution (Dommel,1997). For a typical 500 node problems this repeat solution takes about 0.5s on a CDC 6400. Inserting [λ] thus obtained into Eq. (20) will give the reduced gradient

$$[\nabla r] = \left[\frac{\partial f}{\partial u}\right] + \left[\frac{\partial g}{\partial u}\right]^T[\lambda], \qquad (21)$$

This is the first order sensitivity of the objective function with respect to the control vector [u], with the equality constraints of Eq. (15) rigidly observed

This technique for finding the reduced gradient has been used in expanding an existing power flow solution program into an optimization program at Bonneville Power Administration (Dommel and Tinney, 1998). The basic algorithm is as follows:

1. Guess the control vector [u]

2. Solve the power flow for [x] by Newton's method, with [u] as a fixed parameter. This also yields the Jacobian matrix in factored form which is computationally equivalent to the inverse or transposed inverse (Tinney and Walker, 2001). Coming from step 1, 3 - 4 iterations are required and 1 – 2 when returning from step 6.
3. Solve Eq.(19) for [λ] by a repeat solutions with the factored matrix from step 2.
4. Insert [λ] from step into Eq. (21) to obtain the reduced gradient[∇f].
5. If [∇f] is sufficiently small, the minimum has been reached. Otherwise:
6. Make adjustments on [u]; either with a straightforward optimal gradient technique;

   $$[u^{new}] = \left[u^{old} - \alpha.[\nabla f]\right] \quad (22)$$

   or some other techniques, and return to step 2.

The critical part is step 6. Eq. (22) is one of several possible correction formulas. In Eq. (22); α must be chosen to reach the lowest possible value along the direction of steepest descent -[∇f]. a combination of first-order adjustments of the type of Eq.(22) and second – order adjustments using approximate values for the diagonal elements of the Hessian matrix was finally adopted (Dommel and Tinney, 1998).

To make the optimization realistic, inequality constraints on some parameters of the form

$$u_i \min \leq u_i \leq u_i max \quad (23a)$$

$$x_i \min \leq x_i \leq x_i max \quad (23b)$$

and perhaps functional constraints

$$h_i([x],[u]) \leq 0$$

Must be observed. Eq. (23a) is easily fulfilled by assuring that the adjustment algorithm in step 6 sets u_i to the limit value if it falls outside the permissible region. Its component in the reduced gradient must still be computed in the following adjustment cycles to permit the variable to back off the limits again. Eq. (23b) concerns primarily voltage magnitudes V_i on nodes without voltage control. The introduction of penalty terms of the form

$$w_i = k(x_i - x_{imax})^2 \text{ For } x_i > x_{imax} \quad (24)$$

$$w_i = 0 \text{ Otherwise,}$$

into the objective function was found to be satisfactory for this problem. The factor k is chosen after the first power flow solution to make the sum of the penalties a certain percentage of the objective function. The percentage factor is given by the engineer, which permits him to place more or less emphasis on the constraints of Eq. (23b). The factor k is not changed thereafter and Eq. (23b) will accordingly be violated somewhat when the solution is reached. From an engineering standpoint, a statement of the form $V_i \leq 1.0$ is not meant to be rigid anyhow and a solution of $V_i = 1.01$ may be perfectly acceptable. Fig. 4 shows the result of optimizing the reactive power flow in a normally loaded power system with 328 nodes and 80 components in the control vector[u]; minimization of system losses was achieved with slight increases in penalties. Increasing the factor k in Eq. (24), that is putting more emphasis more emphasis on the inequality constraints Eq. (23b), did not change the solution significantly. The engineer can often define such constraints only roughly as desirable goals; strict observance is not warranted in such cases and would prevent a solution in most cases. Reactive power flow optimization for a light load case (night schedule) in a system with 419 nodes and 122 control variables is shown in Fig. 5. Here, the optimization process reduced primarily the penalty terms, which reflects the physical nature of light load power flows where losses cannot be influenced much and where the major problem is to keep the voltages within acceptance limits.

Optimal power flow solution techniques should also be applicable for optimizations over a period of time, since the component λP_i of the vector [λ] associated with the power equation for P_i in Eq. (18) is directly related to λ and the loss factor in Eq. (11). If the objective function contains no penalty terms, then (Dommel, et al., 2000).

$$\lambda P_i = \lambda(1 - \frac{\partial P_{LOSS}}{\partial P_i}) \quad (25)$$

More research is required, however, before optimization over a period of time becomes practical enough.

CONCLUSION

Traditional dispatch techniques for the optimal operations of power systems have been used for many years. Newer techniques based on nonlinear programming permit overall optimizations, with the power flow solution obtained simultaneously. One such technique, which was found to be practical enough, is described. Other similar techniques have been reported (Sasson *et al.*, 1991) and continued research should lead the way to online applications in the near future.

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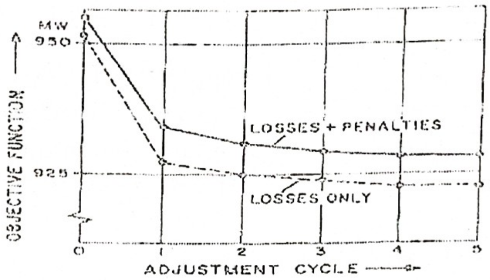
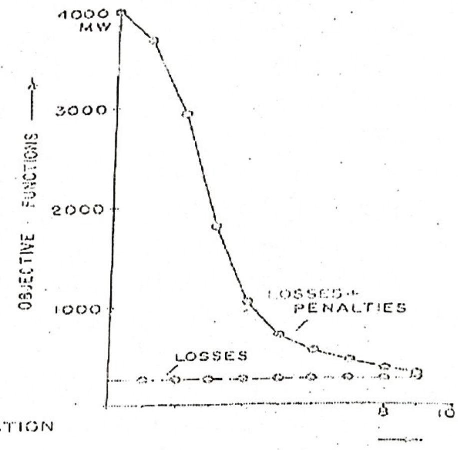


Fig. 4. Optimization of daytime power flow.



TECHNIQUES OF OPTIMIZATION

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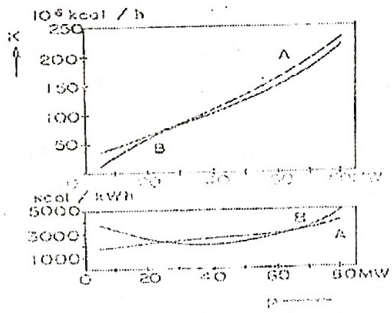

Fig. 1. Costs and incremental costs for units A, B.

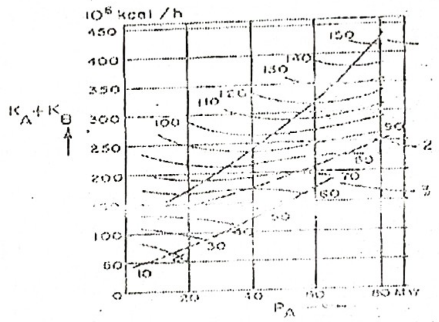

Fig. 2. Total costs as a function of P_A (P_{LOAD} = 10 – 160 MW as parameter)
1 = minima in interior
2 = maxima in interior
3 = minima at boundary.

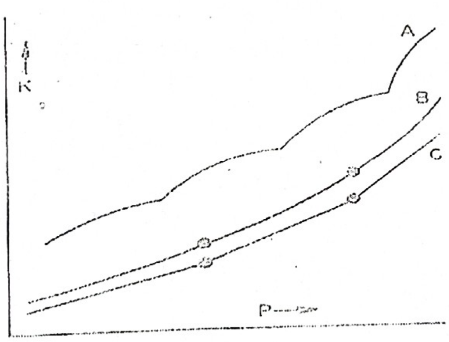

Fig. 3a. Cost curves.

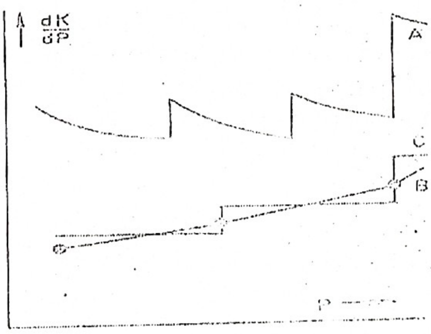

Fig. 3b. Incremental cost curves.