WATER POLLUTION CONTROL

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ABSTRACT

This research dealt with the difficult question of the best treatment plant configuration in a very limited manner where flow of effluent was restricted to flow direct to a discharge point or flow to a single plant and then from the plant to a discharge point. These problems can be handled much more effectively using discrete integer variables in the formulation of the model, as such we switched to the mixed integer, continuous variable formulation. In this model we used a much more extensive and realistic supporting flow network. Quality goals are enforced by introducing the auxiliary variables. The decomposition procedure which consists of alternatively solving the relaxed sub-problems was adopted and found to that these types of regional optimization models would thus seem to offer great promise for regional water quality control.

Keywords: *Optimization, Discharge Point, Treatment Plant, Flow Network, Quality Control, Decomposition, Mathematical Programming, Conservation of Flow*

INTRODUCTION

In an earlier paper (Graves and Hatfield,1972), "Mathematical Programming for Regional Water Quality Management", the model presented dealt with the difficult question of the best treatment plant configuration in a very limited manner. Flow of effluent was restricted to flow direct to a discharge point or flow to a single plant and then from the plant to a discharge point (Sato and Tinny, 2003). There was no flow between plants and hence no way to achieve, say, primary treatment for a large percentage of the effluent at huge, cheap primary plants and subsequent treatment for a much smaller percentage at smaller more expensive secondary and tertiary plants. Since pipe juncture nodes can be considered plants without treatment, the flow pattern was restricted to single juncture points (Dommel and Tinney, 2008). This resulted in practice in manual pipe consolidation and weakened results. The solution procedure for the original model required forcing a solution using only the regional plants. This forces the plants to operate at sufficiently high levels as to utilize their economies of scale. When the other treatment methods are then introduced, they are compelled to complete against the efficiently operating plants (Ogbuoriri *et al* , 1990). Where economically justified they then force out and close the non-competitive plants. This is computationally wasteful as the plants must first be opened and driven down and closed (Sato and Tinny, 2003). There are also difficult problems with infinite derivatives in the cost function with the initial very low or zero flows. These problems can be handled much more effectively using discrete integer variables in the formulation of the model. This possibility was not exploited originally because of the limitations of the solution techniques for large mixed integer and continuous variable nonlinear programming problems (Ogbuoriri *et al.*, 1990). Recent research has greatly expanded our capability to solve integer programming problems. This extended capability coupled with new and more powerful decomposition techniques changes the picture radically. It now seems attractive to switch to the mixed integer, continuous variable formulation.

Mixed Integer Regional Treatment Model

In this model we can use a much more extensive and realistic supporting flow network. This is illustrated by Figure 1. The nodes in the network are of three types. A node is a polluter or source, a sink or a discharge point, or an intermediate node in the network consisting of a pipe juncture or treatment plant. The sources are fixed at the present sites of the identified major polluters. The discharge nodes are the presently used discharge sections of the estuary plus any additional potentially attractive sites. Combining existing and new sites into an optimal total network is particularly nice in this approach because the fixed or capital costs can be separated from the variable or operating costs. This permits a true comparison of the marginal cost of a new plant to an existing plant. The arcs in the network are pipes. They can consist of existing pipes and potentially attractive additional pipes. Again the fixed and variable costs can be separated. Also it is possible to have discrete alternate capacities and enforced choice of pipe size.. Similarly, we can enforce a choice between alternate plant capacities at a given site or between alternate plant sites.

In the mathematical statement of the model we use the following:

Notation

Physical Quantities

A - Matrix of transfer coefficients (mg / ℓ per1b/day)

C - Vector of D.O. goals (mg/ℓ)

r_1 - % Removal of B.O.D. at node 1

q_{ij} - flow from node i to node j (MGD)

j_i - Concentration leaving node i (1b/MG)

m_i - Mass of B.O.D. leaving node i (1b/D)

Costs

f_i- Capital cost of plant I of a given capacity $

v_i- Operating costs of plant I (($/MGD)/% removal)

f_{ij}- Capital cost of pipe between node i and node j of given capacity ($)

v_{ij}- Operating cost of pipe between node i and node j of given capacity ($/MGD)

p_i- Cost of additional treatment at source ($/% removal)

Index sets

R = {j| j is a section or reach of the estuary}

S = {i | node i is a source}

D = {i| node i is a discharge point}

I = {i| node i is not a source or sink}

N= S U D U I set of nodes; a ⊂ N x N set of arcs

a(x) ={y ∈ $N|(x,y) \in a$} nodes "after" x

b(x) = { y ∈ $N|(x,y)$ ∈a} nodes "before" x

Integer Variables

$$z_i = \begin{cases} 0 \; plant\ i\ is\ closed \\ 1 \; plant\ i\ is\ open \end{cases}$$

$$z_{ij} = \begin{cases} 0 \; pipe\ (i,j) is\ closed \\ 1 \; pipe\ (i,j) is\ open \end{cases}$$

The mathematical model is:

Subject to

Conservation of Flow

$$\sum_{j \epsilon a(i)} q_{ij} - \sum_{k \epsilon b(i)} q_{ki} = 0 \qquad i \in x$$

$$\sum_{j \epsilon a(i)} q_{ij} = \bar{q_\iota} \qquad i \in S \; (\bar{q} \text{ current discharge MG/D})$$

Conservation of concentration

$$J_I = \frac{(\sum_{k \epsilon b(i)} q_{ki} J_k)(1-r_i)}{(\sum_{k \epsilon b(i)} q_{ki})} \qquad i \in I \cup D$$

$J_i = \frac{\overline{m}_i}{\overline{q}_i}(1 - r_i)$ $\quad i \in S$ ($\overline{m}_1$ current discharge lb)

Capacity at plant

(ℓ_i –lower capacity, μ_i– upper capacity)

$$z_i . \ell_i \leq \sum_{k\epsilon b(i)} q_{ki} \leq z_i \mu_i$$

(Note that when the plant is closed or not chosen for the network all flow is shut off and when it is opened it functions in a restricted range. By breaking up the range non-convexity can be eliminated and more truly global optimum achieved. This would require merely adding some:

Alternative Constraints (Plants)

$$\sum_{t \in E_i} z_t = 1$$

These constraints would force a choice of only one of the plants in the set E_i)

Capacity of pipes

(ℓ_{ij} –lower, μ_{ij}– upper capacity)

$$Z_{ij} \ell_{ij} \leq q_{ij} \leq Z_{ij} \mu_{ij}$$

(Again the pipes can be eliminated or forced to function in a restricted range. By breaking up the range non-convexity can be eliminated. Again we would employ:

Alternative Constraints (Pipes)

$$\sum_{(t;s) \in E_{ij}} z_{ts} = 1 \quad \text{(Geoffrion, 1972)}$$

These constraints would force a choice of one of the alternatives in the set E_{ij}.)

The quality goals are enforced by introducing the auxiliary variables

$$m_t = J_t (\sum_{k\epsilon b(t)} q_{kt} \quad t \in D$$

$\overline{m}_t$ present discharge in lbs

and the following:

Quality Constraints

$$\sum_{t\epsilon D} a_{it} (m_t - \overline{m}_t) \leq -c_i \quad i \in R$$

The objective is to minimize the total cost function

Minimize $T(Z_t, Z_{ts}, q, r) = PL(Z_t, q, r) + PI(Z_{ts}, q) + PO(r)$

This is broken down into:

Plant Costs

$$PL(Z_t, q, r) = \sum_{t \in I} Z_t \left(f_t + v_t(r) \sum_{k\epsilon b(t)} q_{kt}\right)$$

Pipe Costs

$$PI(Z_{ts}, q) = \sum_{(t,s)\in A} z_{ts} \left(f_{ts} + v_{ts} q_{ts}\right)$$

Polluter Costs

$$PO(r) = \sum_{t \in S} r_t p_t$$

Solution Technique

The foregoing complex mixed integer continuous variable nonlinear programming model presents a truly formidable computational challenge. Although a direct implicit enumeration of the integer variables coupled with the bounding off effect of solving the remaining nonlinear programming problems might be attempted, the outcome of such a venture would be extremely doubtful. Recent advances in Bender's type (see [1]) decomposition offer a much more promising avenue of attack. A Bender's type decomposition applicable to a general nonlinear programming problem

Subject to

$$g^i(y_1, y_2) \leq 0$$

$$g^k(y_2) \leq 0$$

$$\min g^m (y_1, y_2)$$

is possible. In this type of decomposition a subset of the variables, say, the y_2 vector can be used to parameterize a relaxed

Sub-problem

Subject to

$g^i(y_1, \bar{y}_2) \leq 0$

$\min g^m(y_1, \bar{y}_2)$

and the parameters y_2 need not be continuous variables. In the present model the integer variables Z_t and Z_{ts} can be used in this way to yield a supporting network design and a tractable nonlinear programming sub-problem.

Whenever a fixed choice of parameters $\bar{y}_2$ satisfying the remaining constraints

$g^k(\bar{y}_2) \leq 0$

Permits a feasible solution to the sub-problem, we obtain a feasible solution to the complete problem. In order to avoid a random search through plausible choices of $\bar{y}_2$, we employ a class of lower bound functions D. In a stepwise procedure we generate members of this class. By ensuring that at least each lower bound function already generated indicates a better possible solution than the best known incumbent feasible solution we guide the choice of the parameter variables. In point of fact since it is a necessary condition that each lower bound functions is less than the incumbent when there exist a better solution, the lower bound functions provide us with convergence criteria. Neglecting infeasible sub-problems for the most we use the following line of reasoning:

Convergence

Generate stepwise a class D of lower bound functions such that for any

$c^t(y_2) \in D$

and any y_1 such that $y_1.y_2$ is feasible

$c^t(y_2) \leq g^m(y_1, y_2)$

Now attempt to solve the

Master Problem

$g^k(y_2) \leq 0$

$c^t(y_2) \leq g^m(y_1^c, y_2^c) - \varepsilon, t = 1, \dots n$

(Where (y_1^c, y_2^c) is the best known incumbent solution).

If a feasible solution y_2 is obtained we test it in the sub- problem in an attempt to obtain an improved incumbent. If a feasible solution is not obtained in the master problem, then for at least one lower bound function,

$g^m(y_1, y_2) \geq c^t(y_2) > g^m(y_1^c, y_2^c) - \varepsilon$

and $g^m(y_1^c, y_2^c)$ is an ϵ- optimal solution. The identical line of reasoning is employed for infeasible sub-problems where we simply use a violated sub-problem constraint instead of the external function (Hartfield and Graves, 1970). Convergence in a finite number of steps is assured by showing that at most a finite number of $c^t(y_2)$ can be generated between bounded reductions in infeasibility or the external function.

The principal difference between this line of reasoning and the standard Bender's approach is that no attempt is made to obtain the greatest lower bound possible with the finite subset of D already generated (Meyer, Albertson, 1991). This would of course be possible by simply introducing an auxiliary variable y_0 and a master problem of the form

Subject to

$g^k(y_2) \leq 0$

$c^t(y_2) - y_0 \leq 0$

$\min y_0$

There seems no compelling reason to uniformly depress the available lower bound functions as low as possible. In fact this may simply increase the instability and slow down the convergence. In order to generate a class of lower bound functions of interest here we require the two following expansions of the functions.

Expansion1 (with respect to y_1)

$$g^i(y_1, y_2) = g^i(y_1^0, y_2) + \nabla g^i(y_1^0, y_2).\Delta y_1 + \frac{1}{2}\Delta y_1^T H^i(\tilde{y}_1, y_2)\Delta y_1 \qquad (1)$$

Now in order to eliminate the gradients and utilize information furnished by the dual variables in the sub-problem, we must make:

Expansion 2 (with respect to y_2)

$$\nabla g^i(y_1^0, y_2)\Delta y_1 = \nabla g^i(y_1^0, \quad g^i y_2^0)\Delta y_1 + \Delta y_1^T \bar{H}^i(y_1^0, \tilde{y}_2)\Delta y_1 \quad (2)$$

Where

$$\bar{H}^i(y_1^0, \tilde{y}_2) = \frac{\partial^2 g^i(y_1, y_2)}{\partial y_{1k} y_{2j}}$$

is a matrix of mixed partial derivatives. Combining the error terms

$$R^i(\Delta y_1, \Delta y_2) = \frac{1}{2}\Delta y_2^T \bar{H}^i(y_1^0, \tilde{y}_2)\Delta y_1 + \Delta y_1^T \bar{H}^i(y_1^0, \tilde{y}_2)\Delta y_1 \quad (3)$$

yields the desired representation of each function

$$g^i(y_1, y_2) = g^i(y_{1,}^0 y_2) + \nabla g^i(y_1^0, y_2^0).\Delta y_1 + R^i(\Delta y_1, \Delta y_2) \quad (4)$$

Using this representation of our functions and the following duality relation,

$$\textstyle\sum_i X_i \nabla g^i(y_1^0, y_2^0) = \nabla g^m(y_1^0, y_2^0) \quad (5)$$

This is derived as equation (14) of the appendix to "Optimization of a Reverse Osmosis System Using Nonlinear Programming" (see (Hartfield and Graves, 1970,IEEE Report 1991) yields our class of lower bound functions. The argument employed in establishing this class of establishes the "Weak Duality Theorem" of linear programming (Tinney and Walker, 1997). Any ϵ-feasible solution of the sub-problems satisfies

$$g^i(y_1, y_2) = g^i(y_{1,}^0 y_2) + \nabla g^i(y_1^0, y_2^0).\Delta y_1 + R^i(\Delta y_1, \Delta y_2) \leq \epsilon_1 \quad (6)$$

Or

$$\nabla g^i(y_1^0, y_2^0).\Delta y_1 \leq -g^i(y_{1,}^0 y_2) - R^i(\Delta y_1, \Delta y_2) \leq \epsilon_1 \quad (7)$$

Using (5) and (7) we demonstrate that as function of y_2, for any value of y_1

$$g^m(y_1, y_2) = g^m(y_{1,}^0 y_2) + \nabla g^m(y_1^0, y_2^0).\Delta y_1 + R^m(\Delta y_1, \Delta y_2)$$

$$= g^m(y_{1,}^0 y_2) + [\textstyle\sum_i X_i \nabla g^i(y_1^0, y_2^0)]\Delta y_1 + R^m(\Delta y_1, \Delta y_2)$$

$$= g^m(y_{1,}^0 y_2) + \textstyle\sum_i X_i[\nabla g^i(y_1^0, y_2^0)\Delta y_1] + R^m(\Delta y_1, \Delta y_2) \geq g^m(y_{1,}^0 y_2) + \textstyle\sum_i -X_i g^i(y_1^0, y_2) + \textstyle\sum_i(-X_i)R^i(\Delta y_1, \Delta y_2) + R^m(\Delta y_1, \Delta y_2) + \textstyle\sum_i X_i\epsilon_1 \quad (8)$$

For convenience we restate the conditions for a stationary point of the nonlinear algorithm presented in (Hartfield and Graves, 1970, Tinney and Hart 1997).

Local Linear Stationary Point

$\exists$ a set of dual variables $X_p \ni$

1) $\sum_p(-X_p) > B_1$ (Insufficient resolution)

$\exists$ a w and H $\ni$

2) $\sum_{p\in H}(-X_p)g^p(y^0) > -g^w(y^0) - \epsilon$ (Inconsistency)

$\exists$ a $y_0 \in F \ni$

3) $\sum_{i=1}^{m-1}(-X_i)g^i(y^0) > -\epsilon$ (Optimal)

Now choose $\epsilon_1\epsilon/B_1$ and assume that condition 1) of a local linear stationary point is not violated. Then

$\sum_i(-X_i) \geq -B_1$ and $\sum_i X_i\,\epsilon_1 \geq -\epsilon$

And when the error expression

$$\textstyle\sum_i(-X_i)R^i(\Delta y_1, \Delta y_2) + R^m(\Delta y_1, \Delta y_2) \geq 0 \quad (9)$$

We have our lower bound function

$$g^m(y_{1,}^0 y_2) + \textstyle\sum_i(-X_i)g^i(y_1^0, y_2) - \epsilon \leq g^m(y_1, y_2) \quad (10)$$

(Note that in the case where the functions are separable in y_1 and y_2 as in our model, the convexity of the sub-problem in the continuous variables is sufficient to ensure condition (9) and the validity of the decomposition (IEEE Report, 1991). However, we do not have convexity in the sub-problems and a global optimum in our model.)

The decomposition procedure then consists of alternate solving the relaxed sub-problems and the following:

Master Problem

Find a feasible solution to

1) $g^k(y_2) \leq 0$
2) $g^m(y_{1,}^0 y_2) + \sum_i(-X_i)g^i(y_1^0, y_2) - \epsilon \leq g^m(y_1^c, y_2^c) - 2\epsilon$ (Incumbent)
3) $g^w(y_{1,}^0 y_2) + \sum_{p\epsilon H}(-X_p)g^p(y_1^0, y_2) \leq -\epsilon$

Where a constraint of type 2) is added when an optimal solution is obtained for the sub-problem and a constraint of type 3) is added when the sub-problem is inconsistent.

Convergence to an ϵ-optimal solution in a finite number of steps using this decomposition technique is easily demonstrated (George, 1993). Assume we return from a subproblem without achieving a bounded gain in the incumbent solution when we obtained a feasible solution. Then

$$g^m(y_{1,}^0 y_2) \geq g^m(y_1^c, y_2^c) \quad (11)$$

and

$$\sum_i(-X_i)g^i(y_1^0, y_2) > -\epsilon \quad (12)$$

if this set of dual variables is a repeat we contradict

$$g^m(y_{1,}^0 y_2) + \sum_i(-X_i)g^i(y_1^0, y_2) \leq g^m(y_1^c, y_2^c) - \epsilon \quad (13)$$

This has been established from the master problem. Since there are only a finite number of dual solutions between bounded gains in a finite number of steps we exceed any lower bound or fail to solve the master. When we fail to solve the master we have established from one of the lower bound functions that we are within ϵ of the optimum (Meyer and Albertson, 1991).

When we return from the sub-problems with an inconsistent solution we have

$$\sum_{p\epsilon H}(-X_p)g^p(y_1^0, y_2) > -w(y_1^0, y_2) - \epsilon \quad (14)$$

Again if W and H are repeats we contradict

$$g^w(y_{1,}^0 y_2) + \sum_{p\epsilon H}(-X_p)g^p(y_1^0, y_2) \leq -\epsilon \quad (15)$$

This has been established from the master. So in at most a finite number of steps we resolve the inconsistency or demonstrate that the complete problem is inconsistent.

This type of decomposition has recently been employed in a large warehouse location problem (see Geoffrion and Graves, 1999) and proved quite successful results. A full scale computation on semi-realistic data for the Delaware using our earlier model (Graves and Hartfield, 1972) which is similar to the sub-problems has yielded very good results. In the earlier model we solved 80-constraint and 2, 000 variable problems in 15-20 minutes of computation on a 360-91 computer. The solution value of \$2.3 million dollars for 3 mg/ℓ goals compared very favourably with the \$8.3 million dollar cost of the Uniform Treatment at the source policy presently required by the government (George , 1993). These types of regional optimization models would thus seem to offer great promise for regional water quality control Geoffrion, 1972).

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