APPLICATION OF PATTERN RECOGNITION TO SOME PROBLEMS IN ECONOMICS

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ABSTRACT

The choice of a collective decision rule by some outside "advisor" on the basis of various rationality requirements is the classical problem of "individual preference aggregation" in economics. The basic aim of this study is to show how one can use a single underlying structure, viz. that of a preference pattern, and its various mathematical representations to tackle the problem of social choice. The use of classical methods of pattern recognition was used and it led to a completely general concept of collective decision rule and actual processes for collective decision making. We reviewed the aggregation problem for consumer preferences and then translated its formulation in terms of pattern recognition which gave us a very clear understanding of the so-called "voting paradox." Then we related some classical methods of pattern recognition with the motion of a collective decision rule. And finally, we present two possible aggregation algorithms for binary preference patterns. The results thus obtained cast even more light on the relation between pattern recognition and the aggregation problem.

Keywords: *Pattern Recognition, Economics, Consumer, Classical Methods, Algorithm*

INTRODUCTION

Economic theory, as it stands today, displays a rather appalling contrast between the levels of sophistication attained by micro economic theory on the one hand and "public economics" on the other hand. As a matter of fact the very term "public economics" is vague enough to warrant some further specification. Micro economic theory has given as a very general mathematical model of the product such that their consumption (or the consumption of their services) by one individual excludes the consumption of the same goods by another individual. Generally speaking, they are divisible (or, at least, their services are) and they are privately appropriated – at least in most western countries.

A moment's reflection will remind us of the existence of a very different kind of commodity, viz, "public goods." These are usually indivisible commodities, jointly consumed by a large number of individuals who may or may not pay an equal price for their services, depending upon the nature of the good and its particular financing. The provision of public goods is not an individual decision but rather a collective one. The elaborate models of production and consumption of private goods are no longer applicable or to put it another way, the decision rules of utility maximization for the individual consumer and profit maximization for the individual producer are of no help since what we need is a collective decision rule. The study of such collective decision rules is actually two-fold: on the one hand, one may ask how such a rule happens to be- or ought to be- chosen. In other words what rule can we use to decide upon a rule? This problem, which has the flavor of an infinite regress, is known in the literature as the "constitutional choice problem." On the other hand, although the concepts to be outlined below have been quite useful in its solution (see Blin, 2000), we shall concentrate here on a logically posterior problem, viz. the choice of a collective decision rule by some outside "advisor" on the basis of various rationality requirements. This is the classical problem of "individual preference aggregation" in economics. The basic aim of this study is to show how one can use a single underlying structure, viz. that of a preference pattern, and its various mathematical representations to tackle the problem of social choice. The use of classical methods of pattern recognition leads to:

(1) a completely general concept of collective decision rule and

(2) actual processes for collective decision making.

Our study will proceed as follows. First of all, in order to motivate our discussion, we shall briefly review the aggregation problem for consumer preferences and then translate its formulation in terms of pattern recognition which will give us a very clear understanding of the so-called "voting paradox." Then we will relate some classical methods of pattern recognition with the motion of a collective decision rule. And finally, we will present two possible aggregation algorithms for binary preference patterns.

The Aggregation Problem in Economics and the Motion of Consumer Preference Patterns

The "voting paradox": for at least two hundred years it has been known that such a widely used decision rule as majority voting could lead to some peculiar results. More specifically let us consider a group of 3 individuals who are to decide between 3 alternatives (a, b, c), which one they collectively prefer. When we talk about collective preference we must, of course, have decided upon a rule for combining individual preferences. In this case let us suppose that majority voting has been chosen. Let us now consider that each individual is able to rank order the alternatives (a, b, c) and that their respective preference orderings are.

$$\begin{cases} 1 : & b \underset{1}{>} c \underset{1}{>} a \\ 2 : & a \underset{2}{>} b \underset{2}{>} c \\ 3 : & c \underset{3}{>} a \underset{3}{>} b \end{cases}$$

(conventionally, we shall denote the preference relation for any individual h by > which is read".. is preferred by (h) to ...").

These individual preference relations are strictly complete orderings; and as such they satisfy the three basic properties: irreflexivity, asymmetry and transitivity. In other words if A denotes the set of alternatives,

A= {a, b, c, ..., m}

and S the set of individual consumers who are to make a collective decision.

S = {h|h = 1, 2, ..., ϱ}

The individual preference orderings are all:

(i) Irreflexive $<=> \forall i \in A, \forall h \in S: \sim(i > i)$ (where $\sim$ *isread* "it is not the case that" ...).

(ii) Asymmetric $<=> \forall(i,j) \in \{AxA\}, \forall h \in S$:

$(i \overset{>}{h} j) => \sim (j \overset{>}{h} i)$

(iii) Transitive $<=> \forall i,j,k \in A, \forall h \in S$:

$[(I \overset{>}{h} j)(j \overset{>}{h} k)] => (i \overset{>}{h} k)$

Let us now return to the preference system shown in equation (1). The problem is: can we obtain a collective preference ordering in such a society using majority rule. There are $c_3^2 = 3$ paired comparisons to consider: (a, b); (b, c); and (a, c). We have

a > c by a 2/3 majority

b > c by a 2/3 majority

c > a by a 2/3 majority

(Where > without a subscript denotes collective preference). Now if this collective preference is to be an ordering we must have a > c for transitivity to hold. But here we obtain.

a > b > c > a

Which violates transitivity?

It can be verified quite easily that for 3 alternatives another preference system would also result in an intransitive collective preference, namely (a b c), (c b a) and (a c b). This is known as the voting paradox and forms the initial basis of the aggregation problem for consumer preference. Its implication is so obvious that it needs little elaboration: in effect, if a group of individuals, a society makes its social decisions by majority rule it will often be the case that the final winning alternative defeated is then excluded from further consideration, as it is the case in most legislative bodies! Had we decided we would not have been any worse off than under majority voting? But the above presentation of such a paradox fails to illustrate the exact nature of the problem and this is where pattern recognition will first be helpful.

The Concept of Consumer Preference Pattern

In order to discuss the logic of aggregation of individual preferences, we must first characterize each consumer in terms of its own most desired social state which he would choose, if he happened to be the sole decision maker. Put another way, each consumer can be uniquely characterized by an "opinion pattern," in some appropriate feature space. All such opinion pattern can then be compared with each other to detect similarities that will allow us to partition the feature space according into preference classes.

The feature space here will be the space of social states an N-dimensional space where each dimension represents some "issue" in a general sense. For instance it may be the various levels of production of a given public good; as a first approximation each dimension may take on a continuum of values and we may identify our feature space F_N with R^N. On the other hand, it may also take on a finite number of values and in the limit it may even be a two element set {0, 1} for instance. The next step consists in recognizing explicitly the similarity, or dissimilarity that exists between any two preference patterns. To formalize this concept of similarity, we should require that patterns that are "similar" should lie close to each other in some appropriate coordinate systems. This clustering property requires the adoption of some appropriate metrics on the feature space F_N.

We can now summarize the pattern recognition approach to social choice as follows: a set S of ϱ consumer is mapped into specified regions of the feature space F_N – where the features are the public issues at stake. These regions can be viewed as equivalence classes partitioning the F_N space – when the equivalence relation is defined as a similarity index between any two members of a given class. As an illustration for a two-dimensional feature space F_2 let $\{X_1\}$ and $\{X_2\}$ be two such clusters of patterns. This means that all the patterns in $\{X_i\}$ are clustering around the cluster centerX_i^* which may be considered as a representative pattern of the class w_i. The cluster center$X_i^*$for example, can be interpreted as an "average" of the patterns in the class w_i. the separability of these two clusters is guaranteed by the fact that they are non-overlapping. (see figure 1.) On the other hand we may find that some preference patterns are likely to belong to class w_i. What is needed then is some sort of discriminating procedure that will classify any given pattern into one of the classes in F_N. Thus we will use a set of discriminant function $g_i(X)$ which will map an arbitrary preference pattern X into the class w_i if an only if

$$g_i(X) > g_j(X) \forall j \neq i \,; i, j = 1,2, \ldots$$

The decision boundary between class w_i and class w_j will be

$$g_i(X) - g_j(X) = 0.$$

We are then led to search for a class of function $g_i(X)$-which will reach a maximum if and only if pattern x belongs to the class w_i. Let us now see how these familiar concepts of pattern recognition relate to the notion of a collective decision rule.

Discriminate Functions as Social Decision Rules

As we stated at the beginning of this study one of our ultimate goals is to show how the aggregation problem in economics is best understood and solved through the use of a pattern recognition approach. The most startling result, in this respect, consists in showing that the simple majority voting rule can be interpreted as a simple threshold logic unit (TLU).

Majority Voting as a Threshold Logic Unit

Let us consider a vote on a single clear-out issue first. In such a case the feature space F_N is one-dimensional (N=1) and this unique feature can take on only two values {Yes, No} , {-1;+1} say.For each preference pattern the discriminant function is linear:

$$g(x) = (W_h X_h) \text{ where } h=1, \ldots, \varrho.$$

(ϱ = number of consumers)

$$X_h = \begin{cases} -1 \\ +1 \end{cases} \quad \text{and} \quad W_h = +1$$

the social decision rule is then:

$$G(x) = \sin \sum_{h=1}^{\varrho} W_h X_h$$

Where the sin function can take on two values only +1 and -1, and results in indeterminacy if $_{\varrho}$ is an even number and the two pattern classes are of equal size.

The majority voting technique can thus be viewed as a threshold logic unit (TLU) with two possible responses corresponding to the two courses of collective action opened to society in this case.

Needless to say this one- dimensional equal weighting system is but the most primitive and elementary form of decision one could think of its limitation are any. Generally speaking, the one-dimensional restriction will more often than not lead to a "misrepresentation of consumer preference" for it is seldom the case that the issues are independent and can be decided upon sequentially one dimension at a time. The most common case is that a given issue is actually viewed by each consumer as part of complex issues where he takes a global stand by allowing some trade-off between various features of his preference. In cutting down the dimensionality of the feature space to one dimension, we lose some crucial information for pattern classification. We are thus forced to conclude that one of the most fundamental and commonly accepted method of democratic choice, viz. that of proceeding by binary comparisons in order to attain a global multi-valued social decision suffers from a basic drawback: often it will lead to some pathological collective preference pattern-pathological in the same way as the individual patterns. The possible intransitivity of a social pattern obtained by majority voting constitutes such a pathological case.

A new presentation of the paradox of voting can now be achieved to cast some light on this problem. As we know the paradox arises from the sequential application of the majority voting rule to binary issues. In terms of our feature space, each feature is a paired comparison of the form (a vs b) and it has only two values {Yes or No}, {1; 0} say. Thus, each consumer preference pattern is binary and we are supposed to combine these Boolean vectors into some social pattern – which is itself binary. As an example, let there be 3 alternatives, A= {a, b, c} and 3 consumers S= {1, 2, 3}. Hence we need $c_3^2 = 3$ dimensions to compare three alternatives pairwise. If the individual patterns are restricted to complete strict ordering there are only 3! =6 such patterns available, namely.

(a b c)
(c b a)
(b c a)
(c a b)
(b c a)
(a c b)

In terms of Boolean vectors, if the paired comparisons are made in the following order.

a vs b   c vs a   b vs c

We can also write these patterns as

(a b c) < = > (1 0 1)
(c b a) < = > (0 1 0)
(b a c) < = > (0 0 1)
(c a b) < = > (1 1 0)
(b c a) < = > (0 1 1)
(a c b) < = > (1 0 0)

These are the only admissible preference patterns, under the transitivity requirement, but there are exactly 2^N i.e. $2^3 = 8$ possible binary patterns that can arise, the eight vertices of the unit cube in 3 dimensions.

The two excluded patterns are
(0 0 0) < => (a < b < c < a) and
(1 1 1) < =>(a > b > c > a)

They are the origin vertex of the unit cube, and its symmetric along the main diagonal of this cube. As we know the voting paradox will arise in only two cases:

(i) If the preference patterns are all equally distributed in three classes.

$X_1 = (100) <=> a \underset{h}{>} c \underset{h}{>} b$
$X_2 = (010) <=> c \underset{h}{>} b \underset{h}{>} a$
$X_3 = (001) <=> b \underset{h}{>} a \underset{h}{>} c$

Under majority voting the social preference pattern obtained

$X^* = (000) <= (a < b < c < a).$

(ii) If the preference patterns are equally distributed in the three classes.

$= X_1' = (011) <=> b \underset{h}{>} c \underset{h}{>} a$
$= X_2' = (101) <=> a \underset{h}{>} b \underset{h}{>} c$
$= X_3' = (110) <=> c \underset{h}{>} a \underset{h}{>} b$

Under majority voting the social preference pattern is

$X^{**} = (111) = a > b > c > a$

The interpretation of this paradox in terms of linear discriminant function is clear. Let us consider figure 2: each issue, say a vs b is decided upon by drawing a vertical hyper plane H_1parallel to the plane (b vs c;c vs a). the decision rule tells us to choose whichever side of the half space determined by this hyper plane which "contains" most patterns. Similarly we draw hyper planes H_2parallel to the (b vs c; a vs b) plane to decide between c and a; andH_3 parallel to the horizontal plane to decide between b and c. The intersection of these 3 half spaces is a convex region viz. the lower cell on this Figure and the resulting pattern which is the solution to this sequential threshold logic procedure is the point (0 0 0), the origin which is ruled out by the transitivity axiom.
Similarity is society had been of the form (ii), majority voting would have yielded an intransitive pattern, (1 1 1).
Thus, we see that six out of eight patterns (vertices of this cube) are acceptable as social preference pattern and majority voting viewed as a sequential threshold logic unit (hyper plane decision boundaries) may yield any one of the eight patterns.

The general case. The preference patterns displayed by the individuals need not be binary. It is, of course, true that such a presentation more closely conforms with the actual voting techniques encountered in most societies today. But we can, for the sake of generality, assume that each feature displays a continuum of values; and, for instance, we can take as our feature space the N-dimensional Euclidean space.

1.1.1. Let us consider a 2-dimensional feature space F_2 with two pattern classes W_1 and w_2. Let us agree that the sample means X_1 and X_2 constitute a representation pattern for each of the two classes. One possible classified would be a hyperplane normal to the line joining $\overline{X}_1$ and $\overline{X}_2$ and intersecting at mid-distance from $\overline{X}_1$ *and* X_2 in Euclidean space this hyperplane would be represented by the equation

$$g(x) = (\overline{X}_1 - \overline{X}_2) \cdot x + \frac{1}{2}|\overline{X}_2|^2 - \frac{1}{2}|\overline{X}_1|^2 = 0$$

which is of the general form.

$$g(x) = w \cdot x - k$$

where k is a constant.

Here $w = (\overline{X}_1 - \overline{X}_2)$

$$K = -\frac{1}{2}|\overline{X}_2|^2 + \frac{1}{2}|\overline{X}_1|^2.$$

W the weight vector lies in the weight space, the dual of the feature space where x, the pattern vector lies. Intuitively we can give a general policy recommendation for the determination of a vest social pattern to represent the two classes of opinion encountered in such a society. If for instant, the two classes are of the same size, a best pattern would be p*, the point of intersection of the g (x) hyper plane with its weight vector $w = (\overline{X}_1 - \overline{X}_2)$ since of all the points on g (x), only p* minimizes the sum total (Euclidean) distance from g (x) to each element in pattern class w_1 (and w_2)

A special case of a linear classifier is afforded by the minimum distance classifier whereby we use m reference patterns P_1, P_2, P representing the m classes to classify each consumer in one (and only one) of the m pattern classes. For each class W_i the Euclidean distance between an arbitrary pattern X and P_i is of the form

$$d(X) = |x - P_i|^2 = x \cdot x - 2XP_i + P_i \cdot P_i$$

to minimize the distance d_i (x) is equivalent to maximizing the new discriminant function

$$d_i^*(X) = XP_i - \frac{1}{2}P_i - P_j$$

The decision rule then says

$$X \in class w_i <=> d_1^*(x) > d_j^*(x)$$

$$\forall i, j = 1, 2, \ldots, m; \; i \neq j$$

or equivalently : maximize $d_i^*(x)$

$$i \in \{1, 2, \ldots, m\}.$$

There are exactly $c_m^2 = \frac{m(m-1)}{2}$ decision surface (hyperplanes here) needed to partition our feature space into m classes of opinions. Each decision region being formed by the intersection of a finite number of half spaces is itself convex and open – since any pattern on the decision boundary between two regions is classifiable in either one of the two classes. For two public issues (features) and three classes of opinion w_1, w_2 and w_3 an illustration of such a linear classification procedure is given in Figure 3.

The economic interpretation of such classification procedures as the hyper plane technique is clear: it can be viewed as a vote on the public issues at stake – where the vote would actually be a rating and not only a Yes-or-No type of answer. It is clear that many other types of decision rules can be used for classifying preference patterns. For instance, we may wish to consider more than a single representative pattern to describe each class: if we use a point set of representative patterns for each class-exactly analogous to a "council" in voting theory – the minimum distance procedure would yield a piecewise linear decision rule. On the other hand polynomial decision rules, quadric or otherwise, could also be used. Whatever type of classifier we care to choose the complete analogy between the notion of a collective decision rule and discriminant functions as used in pattern recognition still remains.

A Characterization of Pare to Optimal Patterns Space.

An interesting result follows from this presentation of the social choice problem. For many years' welfare economists have used the notion of pare to optimality as one of their least value-restricted tool to rate various "economic states."Among the set E of all possible economic states a sub-set θ is said to form the pare to optimal region if an only if for any state not in θ, there exists some element in θ which is unanimously preferred to the initial state, i.e.

$$\forall\, x \in (E-\theta), \qquad \exists\, x' \in \theta:\ x' \underset{u}{>} x$$

$where \underset{u}{>}$ is read "... unanimously preferred to..."

Thus if we start out from a status quo not in θ, there is always a move that will be unanimously approved. Conditions for achieving pare to-optimality in a private goods economy have long been known. On the other hand, Samuelson (Samuelson, 2004) has derived conditions for pare to optimality in the case of public goods. Within our model if we agree to represent each pattern class by its centroid, the following characterization theorem for pare to optimal pattern can be proved.

Theorem: in a public goods economy represented by the above model, the set of pare to-optimal patterns is the convex closure of the set of centroid representing each pattern class.

Since there is a finite number of such centroids in general, this convex closure is a polyhedron in N-space whose vertices are the centroids P_1, P_2... an illustration for five pattern classes is F_2 is afforded by Figure 4.

If the status quo point I is outside of the convex closure $P_1P_2P_3P_4P_5$ then a move to I_0 say, will be unanimously favored as required for pareto-optimality. * although such a characterization theorem can be extended and serve as a basis for various taxation schemes, this brief discussion was merely meant to point out how useful the general case of preference patterns in a Euclidean feature space could be. However, it appears that the case of binary preference patterns if it may lack some generality, lends itself more directly to some potential applications.

Binary Preference Pattern Aggregation

Binary Patterns as Tournament Matrices. We have previously encountered binary preference patterns in our discussion of the voting paradox (see 3.1 above). At the time we considered them as Boolean vectors but we could also have looked at them as a special class of matrices known as "tournament matrices." If m opponents (alternatives) play each other in a round robin tournament the outcomes can be recorded in matrix form by entering a 1 as the i-jth entry if i defeats j (and a 0 in the j-ith entry). Thus, a tournament $〚matrix〛\_mxm^\wedge T$ is a boolean matrix which verifies:

*for proof and complete discussion of this theorem see (Fu, 2001).

$T + T^{tr} = E - I$ where E is an $\langle mxm \rangle$ matrix of 1' s and I is the $\langle mxm \rangle$ unit matrix.

For m alternatives the set j of all (mxm) tournament matrices will have cardinality $\exp_2 \binom{m}{2}$.

Also, a generalized tournament matrix p is a non-negative (m x m) matrix such that

$\forall i,j \; 0 \leq P_{ij} \leq 1$ and $P + P^{tr} = E - 1$. Such a matrix can arise, for instance, from the assignment of a probability measure Γ on the set J.

Last, we can also note that the upper triangular matrix T is nothing but an binary preference pattern as we defined it earlier since it has_2^m boolean entries written in triangular form. For instance if we have the preference pattern (a b c) and we can write it in binary form as (1 1 1) when the paired comparisons are in the order a vs. b; a vs. c, b vs. c. its tournament matrix representation T is

$$\begin{matrix} & a & b & c \end{matrix}$$

$$T = \begin{matrix} a \\ b \\ c \end{matrix}\begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \text{ and}$$

$$T' = \begin{bmatrix} 1 & 1 \\ & \\ & 1 \end{bmatrix}$$

A natural metric on the space of binary patterns is afforded by hamming distance δ. For instance if two preference patterns T_1' and T_2' are such that.

$$= \underset{(a>b>c)}{T_1'} = \begin{bmatrix} 1 & 1 \\ & 1 \end{bmatrix} \; and \; \underset{(a>c>b)}{T_2'} = \begin{bmatrix} 1 & 1 \\ & 0 \end{bmatrix} \; \delta\,(T_1'\,T_2') = 1$$

In terms of permutation group, it is easy to show (see Duabin, 1998) that the hamming distance between any two patterns is equal to the minimal number of transpositions necessary to transform one pattern (ordering) into the other.

Various solutions of the aggregation problem will now be briefly discussed.

Minimizing the Probability of Misrepresentation of Individual Preference Patterns.

Let us consider $_\varrho$ individual preference patterns $\{T_1 ... T_\varrho\}$- a set of $_\varrho$ tournament matrices, i.e. points in IR^{m2}. Suppose we define the linear aggregate pattern PΣ as the generalized tournament matrix whose entries P_{ij}^{Σ} are the relative frequencies of i > j in this society (other means could be used if we wished, e.g. harmonic, geometric or quadratic means). The fact that this linear aggregate matrix is, in fact a generalized tournament matrix can be shown quite readily (see Moon, 2000).

Now it can also be shown that the set θ of all m x m generalized tournament matrices, a subset of IR^{m^2}, forms a convex polyhedron whose vertices are the elementary tournament matrices T∈J (see Blin, 2000 and Moon, 2000). Define the index set I(T)for any tournament matrix T as:

$$I(T) = \{ i, j) \mid t_{ij} = 1 \}.$$

The probability п (T) that the outcome of a round-robin tournament may be represented by the matrix T_Σ when the a priori probabilities are giver by the matrix P_Σ is then written.

$$\prod(T) = \quad (i,j) \in I(T) \; P_{ij}^{\Sigma}$$

The error probability (e) is then: e = 1 - ∏ (T). To minimize e we must minimize ∏ (T). It can be shown (see Blin, 2000) that the optimal solution leads to the following decision rule:

$$\text{Choose} \begin{cases} P_{ij} \; if \; and \; only \; if \; P_{ij} > P_{ji} \\ P_{ji} \; if \; and \; only \; if \; P_{ji} > P_{ij} \end{cases}$$

(In the case where $P_{ij} = P_{ji}$ we have indeterminacy.)

This result is the maximum likelihood decision rule which is thus seen to be identical to the majority decision rule in voting theory. In this way we can give a decision theoretic rationale for the majority voting rule. *

However, even though we started, by assumption, from individual transitive patterns, we end up with a social pattern that is optimal in the sense of minimizing the misrepresentation probability but is not necessarily transitive.

*In (Duabin, 1998) it is also shown that another algorithm, the "minimal distance algorithm," can be devised, using the geometric properties of the set of binary patterns viewed as tournament matrices.

Generating an Aggregate Transitive Preference Pattern.

We can now use the hamming metric defined on the binary patterns ($P_1, ... P_\ell$). The aggregation problem can be stated: find P*, a transitive pattern which

$$\text{Min} \sum_{h=1}^{\ell} d\,(P_h - P^*).$$

This procedure can be justified by noting that if we assume (1) that individual disutility is an increasing function of the distance between the social pattern and the preferred pattern of each individual and (2) that disutility's are interpersonally comparable, then minimizing the total distance index is equivalent to minimizing social loss.

A solution algorithm can be outlined as follows:

1. Find all pairs (i, j) such that $i \underset{h}{>} j \ \forall h$ and take the transitive closure of this partial order.
2. If P* is a total order go to 9
3. List all pairs (i, j) of quasi-agreement in order of decreasing agreement.
4. Include pair ≠× from this list.
5. Check for transitivity in the resting partial order P*.
6. If P* is intransitive, exclude pair ≠×.
7. If P* is a total order go to 9.
8. Go to 4
9. Stop.

Example.
Let A = {a, b, c, d} and $\ell = 5$
P_1 = (b, d, c, a)
P_2 = (a, d, b, c)
P_3 = (a, c, b, d)
P_4 = (c, d, b, a)
P_5 = (b, a, d, c)

The computation can be summarized in the table below.

	a vs b	a vs c	a vs d	b vs c	b vs d	c vs d	$d(P_h, P^*)$
b d c a	0	0	0	1	1	0	4
a d b c	1	1	1	1	0	0	2
a c b d	1	1	1	1	1	0	1
c d b a	0	0	0	0	0	0	5
b a d c	0	1	1	1	1	0	2
number of patterns in agreement	2	3	3	4	3	1	Σ= 16
P*	1	1	1	1	1	1	

CONCLUSION

This brief discussion of two possible aggregation processes was meant to suggest how useful the concept of binary preference pattern could be. Many other algorithms can and have in fact been devised, starting from other optimality criteria and other algebraic structures (see Moon, 2000). Moreover, the search for a class of computationally "economical" algorithms has been quite successful so far as will be shown elsewhere – and it suffices to note, here, that the results thus obtained cast even more light on the relation between pattern recognition and the aggregation problem.

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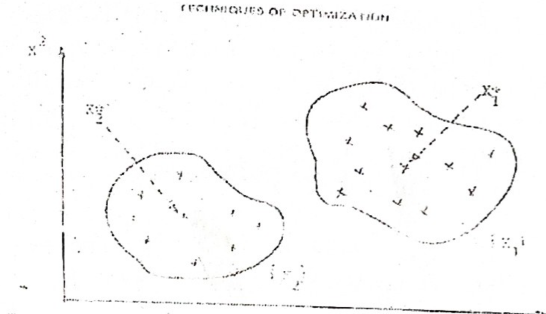

Figure 1. Pattern classes in a two-dimensional feature space.

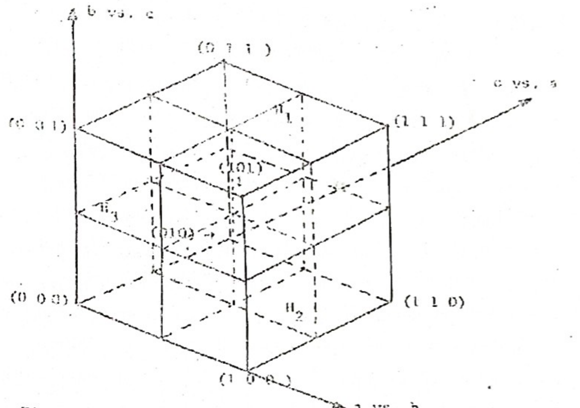

Figure 2. A geometric illustration of the majority voting

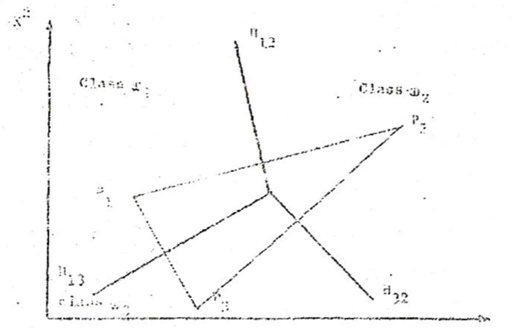

Figure 3. Minimum distance classification for 3 classes.

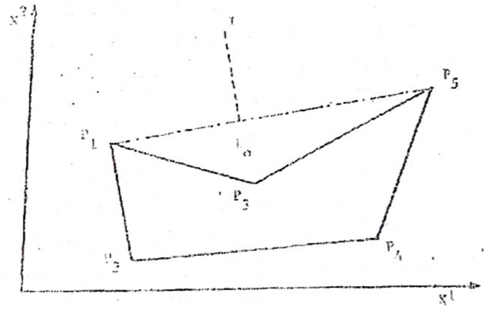

Figure 4. Set of Pareto optimal patterns for a five-class society in F_2.